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Department of Mathematics KDK College of Engineering, Nagpur in Association with NUMTA is organizing a three days International Conference on 4th,5th & 6th October 2021. The conference aims to bring together academicians, researchers and experts from various fields of Mathematical Sciences to interact and exchange their views for supporting professional development.

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CONTENTS

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Formulation of Solutions of a Special Standard Quadratic Congruence modulo a Prime Multiple of Powered Even Prime

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ABSTRACT

In this manuscript, the authors have considered a very special type of standard quadratic congruence of composite modulus modulo a prime multiple of a powered even prime integer for formulation of its solutions. A simple formulation is established and formulated. It finds the solutions very easily with less effort. The method proved time-saving. Formulation is the merit of the paper.

KEY- WORDS : Even Prime, Formulation, Prime Multiple, Quadratic Congruence.

I. INTRODUCTION

The Congruence is a topic in the book of Number Theory. Discussions on Linear and quadratic congruence are found in those books.

A standard quadratic congruence is defined mathematically as: $x^2 \equiv a \pmod{m}$ and the values of x that satisfies the congruence are called solutions of it i.e.to find those values of x, whose square when divided by m, gives the remainder as a.

In this paper, some special types of a & m are considered i.e.$a=2^n$, $m=2^n.p$, p being an odd prime positive integer. Then the congruence under consideration becomes:

$$x^2 \equiv 2^n \pmod{2^n.p}.$$

PROBLEM-STATEMENT

Here the problem of study is:

To find the solutions of the congruence $x^2 \equiv 2^n \pmod{2^n.p}$;p an odd prime; n is even positive integer with $n=2m$.

II. LITERATURE REVIEW

The present standard quadratic congruence considered for formulation of its solutions is neither be formulated nor discussed anywhere in the literature of mathematics. Zuckerman et el [1], Thomas Koshy [2], David M

Burton [3] all have discussed the standard quadratic congruence of prime and composite modulus but nothing is found reported for the present problem. The author has already formulated many such standard quadratic congruence of composite modulus [4], [5], [6], [7]. In continuation, the author with his co-author have consider the problem under consideration for formulation of solutions.

III. ANALYSIS & RESULTS

Consider the congruence: $x^2\equiv 2^n \pmod{2^n.p}$;p an odd prime; n is even positive integer with n=2m.
Then it reduces to the form: $x^2\equiv 2^{2m} \pmod{2^n.p}$.
For its solutions, let $x\equiv 2^m.pk\pm 2^m \pmod{2^n.p}$
Then, $x^2\equiv (2^m.pk\pm 2^m)^2 \pmod{2^n.p}$
$\equiv (2^m.pk)^2\pm 2.2^m.pk.2^m+2^{2m} \pmod{2^n.p}$
$\equiv 2^m\, pk(2^m.pk\pm 2.2^m)+2^{2m} \pmod{2^n.p}$
$\equiv 2^m\, pk.2^m\,(pk\pm 2)+2^{2m} \pmod{2^n.p}$
$\equiv 2^{2m}.pk(pk\pm 2)+2^{2m} \pmod{2^n.p}$
$\equiv 2^n\, pk(pk\pm 2)+2^{2m} \pmod{2^n.p}$
$\equiv 0+2^{2m} \pmod{2^n.p}$
$\equiv 2^{2m} \pmod{2^n.p}$
Therefore, the formulation satisfies the congruence and hence it can be considered as solutions of the said congruence for different values of positive integer k.
But for $k=2^m$, the solutions formula reduces to the form:
$x\equiv 2^m.p.2^m\pm 2^m \pmod{2^n.p}$
$\equiv 2^{2m}.p\pm 2^m \pmod{2^n.p}$
$\equiv 2^n.p\pm 2^m \pmod{2^n.p}$
$\equiv 0\pm 2^m \pmod{2^n.p}$
These are the same solutions as for k=0.
Also, for $k=2^m+1$, the solutions formula reduces to the form:
$x\equiv 2^m.p.(2^m+1)\pm 2^m \pmod{2^n.p}$
$\equiv 2^{2m}.p+2^m.p\pm 2^m \pmod{2^n.p}$
$\equiv (0+2)^m.p\pm 2^m \pmod{2^n.p}$
$\equiv 2^m.p\pm 2^m \pmod{2^n.p}$
These are the same solutions as for k=1.
Therefore, all the solutions are given by
$x\equiv 2^m.pk\pm 2^m \pmod{2^n.p};k=0,1,2,\cdots\cdots,(2)^m-1)$.
This gives $(2.2)^m=2^{(m+1)}$ incongruent solutions as for each value of k gives exactly two solutions.

ILLUSTRATIONS

Example-1: Consider the congruence $x^2\equiv 2^{6} \pmod{2^6.5}$
It can be written as: $x^2\equiv 2^{2.3} \pmod{2^6.5}$
It is of the type $x^2\equiv 2^{2m} \pmod{2^n.p}$ with m=3,n=6,p=5.

It has exactly $2^{(m+1)}$ incongruent solutions.
Its solutions are given by
$x \equiv 2^m.pk \pm 2^m \pmod{2^n.p}$
$\equiv 2^3.5k \pm 2^{(3)} \pmod{2^6.5}$
$\equiv 2^3.5k \pm 2^{(3)} \pmod{2^6.5}$
$\equiv 40k \pm 8 \pmod{320}; k=0,1,2,3,4,5,6,7.$
$\equiv 0 \pm 8; 40 \pm 8; 80 \pm 8; 120 \pm 8; 160 \pm 8; 200 \pm 8; 240 \pm 8; 280 \pm 8 \pmod{320}$
$\equiv 8, 312; 32, 48; 72, 88; 112, 128; 152. 168; 192, 208; 232, 248; 272, 288 \pmod{320}$
These are the sixteen solutions.
Example-2: Consider the congruence $x^2 \equiv 2^{(8)} \pmod{2^8.3}$
It can be written as: $x^2 \equiv 2^{(2.4)} \pmod{2^8.3}$
It is of the type $x^2 \equiv 2^{(2m)} \pmod{2^n.p}$ with $m=4, n=8, p=3$.
Its solutions are
$x \equiv 2^m.pk \pm 2^m \pmod{2^n.p}$
$\equiv 2^4.3k \pm 2^{(4)} \pmod{2^8.3}$
$\equiv 2^4.3k \pm 2^{(4)} \pmod{2^8.3}$
$\equiv 48k \pm 16 \pmod{1280}; k=0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15.$
$\equiv 0 \pm 8; 48 \pm 8; 96 \pm 8; 144 \pm 8; 192 \pm 8; 240 \pm 8; 288 \pm 8; 336 \pm 8;$
$384 \pm 16; 432 \pm 16; 480 \pm 16; 528 \pm 16; 576 \pm 16; 624 \pm 16; \pmod{1280}$
$\equiv 8, 312; 32, 48; 72, 88; 112, 128; 152. 168; 192, 208; 232, 248; 272, 288 \pmod{1280}$
These are the thirty two solutions.

IV. CONCLUSION

Therefore, it is concluded that the congruence under consideration:
$x^2 \equiv 2^{2m} \pmod{2^{2m}.p}$ has $2.2^m = 2^{(m+1)}$ incongruent solutions given by
$x \equiv 2^m.pk \pm 2^m \pmod{2^{2m}.p}; k=0,1,2,\cdots\cdots,(2^m-1).$

Merit of the paper

The quadratic congruence considered for formulation are successfully formulated and the formulation help the authors to find the solutions very easily. It made the study of congruence easy. These are the merit of the paper.

Future Scope

The current paper has a future scope of formulation of solutions. Another standard quadratic congruence of same modulus can be considered as a problem of study.

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V. REFERENCES

[1] . Prof. Thomas Koshy, 2009, Elementary Number Theory with Applications, Academic Press, Second Edition, Indian print, New Dehli, India, ISBN: 978-81-312-1859-4.

[2] . Zuckerman at el, An Introduction to The Theory of Numbers, fifth edition, Wiley India (P) Ltd, 2008, ISBN: 978-81-265-1811-1.

[3] . Burton David M., Elementary Number Theory, seventh edition, Mc Graw Hill education (India), 2017. ISBN: 978-1-25-902576-1.

[4] . B M Roy, A Study on Standard Quadratic Congruence of Prime Modulus having Solutions Consecutive-Integers, (JETIR),ISSN: 2349-5162, Vol-08, Issue-07, July-21.

[5] . B M Roy, A A Qureshi, Formulation of Solutions of a Special Standard Quadratic Congruence modulo an Even Prime Integer raised to the power n, International Journal of Physics and Mathematics (IJPM), ISSN: 2664-8644, Vol-03, Issue-02, Aug-21.

[6] . B M Roy, Formulation of Standard Quadratic Congruence of Even Composite Modulus modulo an even prime raised to the power n, International Journal of Research in Applied Science & Engineering Technology (IJRASET), ISSN: 2321-9653, Vol-09, Issue-09, Aug-21.

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Common Fixed Point Theorem for Six Weakly Compatible Mappings Satisfying Generalized Contractive Condition of Integral Type

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ABSTRACT

We prove some unique common fixed point result for three pairs of weakly compatible mappings satisfying a generalized contractive condition of Integral type in complete G-metric space. The present theorem is the extension of many results already existing in the literature.

Keywords : Fixed point, Complete G- metric space, G-Cauchy sequence, weakly compatible mapping, and Integral Type contractive condition.

I. INTRODUCTION

Generalization of Banach contraction principle in various ways has been studied by many authors. One may refer Beg I. & Abbas M.[2] , Dutta P.N. & Choudhury B.S.[3] ,Khan M.S., Swaleh M. & Sessa, S.[9] , Rhoades B.E.[12] , Sastry K.P.R. & Babu G.V.R.[13] , Suzuki T.[15] . Alber Ya.I. & Guerre-Delabriere S. [1] had proved results for weakly contractive mapping in Hilbert space , the same was proved by Rhoades B.E.[12] in complete metric space.

Jungck G.[6] proved a common fixed point theorem for commuting mappings which is the extension of Banach contraction principle. Sessa S.[14] introduced the term "Weakly commuting mappings" which was generalized by Jungck G.[6] as "Compatible mappings". Pant R.P.[11] coined the notion of "*R*-weakly commuting mappings", whereas Jungck G.& Rhoades B.E. [8] defined a term called "weakly compatible mappings" in metric space.

Fisher B. [4] proved an important Common Fixed Point theorem for weakly compatible mapping in complete metric space.

Mustafa in collaboration with Sims [10] introduced a new notation of generalized metric space called G- metric space in 2006. He proved many fixed point results for a self-mapping in G- metric space under certain conditions.

Now we give some preliminaries and basic definitions which are used through-out the paper.

Definition 1.1: Let X be a non empty set, and let $G : X \times X \times X \to R^{+}$ be a function satisfying the following properties:

(G_1) $G(x, y, z) = 0$ if $x = y = z$

(G_2) $0 < G(x, x, y)$ for all $x, y \in X$,with $x \neq y$

(G_3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$, with $y \neq z$

(G_4) $G(x, y, z) = G(x, z, y) = G(y, z, x)$ (Symmetry in all three variables)

(G_5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, for all $x, y, z, a \in X$ (rectangle inequality)

Then the function G is called a generalized metric space, or more specially a G- metric on X, and the pair (X, G) is called a G–metric space.

Definition 1.2: Let (X, G) be a G - metric space and let $\{x_n\}$ be a sequence of points of X , a point $x \in X$ is said to be the limit of the sequence $\{x_n\}$, if $\lim_{n,m \to +\infty} G(x, x_n, x_m) = 0$, and we say that the sequence $\{x_n\}$ is G - convergent to x or $\{x_n\}$ G -converges to x.

Thus, $x_n \to x$ in a G - metric space (X, G) if for any $\in > 0$ there exists $k \in N$ such that $G(x, x_n, x_m) < \in$, for all $m, n \geq k$

Proposition 1.3: Let (X, G) be a G - metric space. Then the following are equivalent:

i) $\{x_n\}$ is G - convergent to x

ii) $G(x_n, x_n, x) \to 0$ as $n \to +\infty$

iii) $G(x_n, x, x) \to 0$ as $n \to +\infty$

iv) $G(x_n, x_m, x) \to 0$ as $n, m \to +\infty$

Proposition 1.4 : Let (X, G) be a G - metric space. Then for any x, y, z, a in X it follows that

i) If $G(x, y, z) = 0$ then $x = y = z$

ii) $G(x, y, z) \leq G(x, x, y) + G(x, x, z)$

iii) $G(x, y, y) \leq 2G(y, x, x)$

iv) $G(x, y, z) \leq G(x, a, z) + G(a, y, z)$

v) $G(x, y, z) \leq \frac{2}{3}\begin{pmatrix} G(x, y, a) + G(x, a, z) \\ + G(a, y, z) \end{pmatrix}$

vi) $G(x, y, z) \leq \begin{pmatrix} G(x, a, a) + G(y, a, a) \\ + G(z, a, a) \end{pmatrix}$

Definition 1.5: Let (X, G) be a G - metric space. A sequence $\{x_n\}$ is called a G - Cauchy sequence if for any $\in > 0$ there exists $k \in N$ such that $G(x_n, x_m, x_l) < \in$ for all $m, n, l \geq k$, that is $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to +\infty$.

Proposition 1.6: Let (X,G) be a G - metric space .Then the following are equivalent:

i) The sequence $\{x_n\}$ is G - Cauchy;

ii) For any $\in > 0$ there exists $k \in N$ such that $G(x_n, x_m, x_m) < \in$ for all $m, n \geq k$

Proposition 1.7: A G - metric space (X,G) is called G -complete if every G -Cauchy sequence is G - convergent in (X,G).

Proposition 1.8: Let (*X*, G) be a G- metric space. Then the function G(*x*, *y*, *z*) is jointly continuous in all three of its variables.

Definition 1.9 : Let f and g be two self – maps on a set X. Maps f and g are said to be commuting if $fgx = gfx$, for all $x \in X$

Definition 1.10 : Let f and g be two self – maps on a set X. If $fx = gx$, for some $x \in X$ then x is called coincidence point of *f* and *g.*

Definition 1.11: Let f and g be two self – maps defined on a set X , then f and g are said to be weakly compatible if they commute at coincidence points. That is if $fu = gu$ for some $u \in X$, then $fgu = gfu$.

The main aim of this paper is to prove a unique common fixed point theorem for three pairs of weakly compatible mappings satisfying Integral type contractive condition in a complete G – metric space.

The result may be the extension of some results already existing in the literature as in the present paper we are considering six compatible mappings in G-metric space.

II. MAIN RESULT

Theorem 2.1 : Let (X,G) be a complete G-metric space and $L, M, N, P, Q, R : X \to X$ be mappings such that

i) $L(X) \subset P(X)$, $M(X) \subset Q(X)$, $N(X) \subset R(X)$

ii) $$\xi\left\{\int_0^{G(Lx,My,Nz)} f(t)\,dt\right\} \leq \xi\left\{\int_0^{G(Px,Qy,Rz)} f(t)\,dt\right\} - \eta\left\{\int_0^{G(Px,Qy,Rz)} f(t)\,dt\right\}$$

(2.1.1)

for all $x, y, z \in X$ where $\xi : [0,\infty) \to [0,\infty)$ is a continuous and non-decreasing function ,

$\eta : [0,\infty) \to [0,\infty)$ is a lower semi continuous and non-decreasing function such that

$\xi(t) = \eta(t) = 0$ if and only if $t = 0$, also $f : [0,\infty) \to [0,\infty)$ is a Lebesgue integrable function

which is summable on each compact subset of R^+ , non negative and such that for each $\in > 0$,

$$\int_0^{\in} f(t)\,dt > 0$$

iii) The pairs (L,P) , (M,Q) , (N,R) are weakly compatible.

Then L,M,N,P,Q,R have a unique common fixed point in X.

Proof : Let x_0 be an arbitrary point of X and define the sequence $\{x_n\}$ in X such that

$y_n = Lx_n = Px_{n+1}$, $y_{n+1} = Mx_{n+1} = Qx_{n+2}$,

$y_{n+2} = Nx_{n+2} = Rx_{n+3}$

Consider , $\xi\left\{\int_0^{G(y_n,y_{n+1},y_{n+2})} f(t)\,dt\right\} = \xi\left\{\int_0^{G(Lx_n,Mx_{n+1},Nx_{n+2})} f(t)\,dt\right\}$

$$\le \xi\left\{\int_0^{G(Px_n,Qx_{n+1},Rx_{n+2})} f(t)\,dt\right\} - \eta\left\{\int_0^{G(Px_n,Qx_{n+1},Rx_{n+2})} f(t)\,dt\right\}$$

$$= \xi\left\{\int_0^{G(y_{n-1},y_n,y_{n+1})} f(t)\,dt\right\} - \eta\left\{\int_0^{G(y_{n-1},y_n,y_{n+1})} f(t)\,dt\right\} \qquad (2.1.2)$$

$$\le \xi\left\{\int_0^{G(y_{n-1},y_n,y_{n+1})} f(t)\,dt\right\}$$

Since ξ is continuous and has a monotone property ,

$$\therefore \int_0^{G(y_n,y_{n+1},y_{n+2})} f(t)\,dt \le \int_0^{G(y_{n-1},y_n,y_{n+1})} f(t)\,dt \qquad (2.1.3)$$

Let us take $\delta_n = \int_0^{G(y_n,y_{n+1},y_{n+2})} f(t)\,dt$, then it follows that δ_n is monotone decreasing and lower bounded sequence of numbers.

Therefore there exists $k \ge 0$ such that $\delta_n \to k$ as $n \to \infty$. Suppose that $k > 0$

Taking limit as $n \to \infty$ on both sides of (2.1.2) and using that η is lower semi continuous , we get , $\xi(k) \le \xi(k) - \eta(k) < \xi(k)$, which is a contradiction. Hence $k = 0$.

This implies that $\delta_n \to 0$ as $n \to \infty$ i.e.

$$\int_0^{G(y_n,y_{n+1},y_{n+2})} f(t)\,dt \to 0 \text{ as } n \to \infty. \qquad (2.1.4)$$

Now , we prove that $\{y_n\}$ is a G- Cauchy sequence. On the contrary , suppose it is not a G- Cauchy sequence.

$\therefore$ There exists $\in > 0$ and sub sequences $\{y_{m(i)}\}$ and $\{y_{n(i)}\}$ such that for each positive integer i, $n(i)$ is minimal in the sense that , $G(y_{n(i)}, y_{m(i)}, y_{m(i)}) \ge \in$ and $G(y_{n(i-1)}, y_{m(i)}, y_{m(i)}) < \in$

Now , $$\in \le G(y_{n(i)}, y_{m(i)}, y_{m(i)}) \le G(y_{n(i)}, y_{m(i-1)}, y_{m(i-1)}) + G(y_{m(i-1)}, y_{m(i)}, y_{m(i)})$$

$$< \in + G(y_{m(i-1)}, y_{m(i)}, y_{m(i)}) \qquad (2.1.5)$$

Let $0 < \alpha = \int_0^{\epsilon} f(t)\,dt \le \int_0^{G(y_{n(i)},y_{m(i)},y_{m(i)})} f(t)\,dt \le \int_0^{\epsilon + G(y_{m(i-1)},y_{m(i)},y_{m(i)})} f(t)\,dt$

Taking $i \to \infty$, and using (2.1.4) , we get , $\lim_{i\to\infty} \int_0^{G(y_{n(i)},y_{m(i)},y_{m(i)})} f(t)\,dt = \alpha$ (2.1.6)

Now , using rectangular inequality , we have

$$G(y_{n(i)},y_{m(i)},y_{m(i)}) \le G(y_{n(i)},y_{n(i-1)},y_{n(i-1)}) + G(y_{n(i-1)},y_{m(i-1)},y_{m(i-1)}) + G(y_{m(i-1)},y_{m(i)},y_{m(i)}) \quad (2.1.7)$$

$$G(y_{n(i-1)},y_{m(i-1)},y_{m(i-1)}) \le G(y_{n(i-1)},y_{n(i)},y_{n(i)}) + G(y_{n(i)},y_{m(i)},y_{m(i)}) + G(y_{m(i)},y_{m(i-1)},y_{m(i-1)}) \quad (2.1.8)$$

$$\therefore \int_0^{G(y_{n(i)},y_{m(i)}y_{m(i)})} f(t)\,dt \le \int_0^{G(y_{n(i)},y_{n(i-1)},y_{n(i-1)}) + G(y_{n(i)},y_{m(i-1)},y_{m(i-1)}) + G(y_{m(i-1)},y_{m(i)},y_{m(i)})} f(t)\,dt$$

and

$$\int_0^{G(y_{n(i-1)},y_{m(i-1)},y_{m(i-1)})} f(t)\,dt \le \int_0^{G(y_{n(i-1)},y_{n(i)},y_{n(i)}) + G(y_{n(i)},y_{m(i)},y_{m(i)}) + G(y_{m(i)},y_{m(i-1)},y_{m(i-1)})} f(t)\,dt$$

Taking limit as $i \to \infty$ and using (2.1.4) , (2.1.6) we get

$$\int_0^{G(y_{n(i-1)},y_{m(i-1)},y_{m(i-1)})} f(t)\,dt \le \alpha \le \int_0^{G(y_{n(i-1)},y_{m(i-1)},y_{m(i-1)})} f(t)\,dt$$

This implies that , $\lim_{i\to\infty} \int_0^{G(y_{n(i-1)},y_{m(i-1)},y_{m(i-1)})} f(t)\,dt = \alpha$ (2.1.9)

Now , from (2.1.1) , we have ,

$$\xi\left\{\int_0^{G(y_{n(i)},y_{m(i)},y_{m(i)})} f(t)\,dt\right\} \le \xi\left\{\int_0^{G(y_{n(i-1)},y_{m(i-1)},y_{m(i-1)})} f(t)\,dt\right\} - \eta\left\{\int_0^{G(y_{n(i-1)},y_{m(i-1)},y_{m(i-1)})} f(t)\,dt\right\}$$

$\therefore$ Taking limit as $i \to \infty$ and using (2.1.6) , (2.1.8) we will have , $\xi(\alpha) \le \xi(\alpha) - \eta(\alpha) < \xi(\alpha)$

which is a contradiction. Hence we have $\alpha = 0$.

Hence $\{y_n\}$ is a G- Cauchy sequence. Since (X, G) is a complete G-metric space , there exists a point $u \in X$ such that $\lim_{n\to\infty} y_n = u$

i.e. $\lim_{n\to\infty} Lx_n = \lim_{n\to\infty} Px_{n+1} = u$,

$\lim_{n\to\infty} Mx_{n+1} = \lim_{n\to\infty} Qx_{n+2} = u$, $\lim_{n\to\infty} Nx_{n+2} = \lim_{n\to\infty} Rx_{n+3} = u$

As $Lx_n \to u$ and $Px_{n+1} \to u$, therefore we can find some $h \in X$ such that $Qh = u$.

$$\therefore \begin{aligned}&\xi\left\{\int_0^{G(Lx_n,Mh,Mh)} f(t)\,dt\right\}\le \xi\left\{\int_0^{G(Lx_n,Mh,Nx_{n+1})} f(t)\,dt\right\}\\&\le \xi\left\{\int_0^{G(Px_n,Qh,Rx_{n+1})} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Px_n,Qh,Rx_{n+1})} f(t)\,dt\right\}\end{aligned}$$

On taking limit as $n\to\infty$, we get ,

$$\xi\left\{\int_0^{G(u,Mh,Mh)} f(t)\,dt\right\}\le \xi(0)-\eta(0)$$

$\therefore\ \xi\left\{\int_0^{G(u,Mh,Mh)} f(t)\,dt\right\}=0$, which implies that $Mh=u$.

Hence $Mh=Qh=u$ i.e. h is the point of coincidence of M and Q.

Since the pair of maps M and Q are weakly compatible , we write $MQh=QMh$ i.e. $Mu=Qu$.

Also , $Mx_{n+1}\to u$ and $Qx_{n+2}\to u$, $\therefore$ we can find some $v\in X$ such that $Pv=u$.

$$\therefore \begin{aligned}&\xi\left\{\int_0^{G(Lv,Mx_{n+1},Mx_{n+1})} f(t)\,dt\right\}\le \xi\left\{\int_0^{G(Lv,Mx_{n+1},Nx_{n+2})} f(t)\,dt\right\}\\&\le \xi\left\{\int_0^{G(Pv,Qx_{n+1},Rx_{n+2})} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Pv,Qx_{n+1},Rx_{n+2})} f(t)\,dt\right\}\end{aligned}$$

On taking limit as $n\to\infty$, we get ,

$$\xi\left\{\int_0^{G(Lv,u,u)} f(t)\,dt\right\}\le \xi(0)-\eta(0)$$

$\therefore\ \xi\left\{\int_0^{G(Lv,u,u)} f(t)\,dt\right\}=0$, which implies that $Lv=u$. Hence we have $Lv=Pv=u$ i.e. v is the point of coincidence of L and P. Since the pair of maps L and P are weakly compatible , we can write $LPv=PLv$ i.e. $Lu=Pu$.

Again , $Nx_{n+2}\to u$ and $Rx_{n+3}\to u$, therefore we can find some $w\in X$ such that $Rw=u$.

$$\therefore\ \xi\left\{\int_0^{G(Lx_n,Mx_{n+1},Nw)} f(t)\,dt\right\}\le \xi\left\{\int_0^{G(Px_n,Qx_{n+1},Rw)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Px_n,Qx_{n+1},Rw)} f(t)\,dt\right\}$$

On taking limit as $n\to\infty$, we get ,

$$\xi\left\{\int_0^{G(u,u,Nw)} f(t)\,dt\right\}\le \xi(0)-\eta(0)$$

i.e. $\xi\left\{\int_0^{G(u,u,Nw)} f(t)\,dt\right\}=0$, which implies that $Nw=u$.

Thus we get $Nw=Rw=u$ i.e. w is the coincidence point of N and R.

Since the pair of maps N and R are weakly compatible , we have $NRw=RNw$ i.e. $Nu=Ru$

Now , we show that u is the fixed point of L.

Consider , $\xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}=\xi\left\{\int_0^{G(Lu,Mh,Nw)} f(t)\,dt\right\}\leq\xi\left\{\int_0^{G(Pu,Qh,Rw)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Pu,Qh,Rw)} f(t)\,dt\right\}$

$\therefore\ \xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}\leq\xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}$

i.e. $\xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}\leq\xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}$

i.e. $\xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}<\xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}$, which is a contradiction. $\therefore$ we get $Lu=u$

$\therefore\ Lu=Pu=u$ i.e. u is fixed point of L and P.

Now , we prove that u is fixed point of M.

$$\xi\left\{\int_0^{G(u,u,Mu)} f(t)\,dt\right\}=\xi\left\{\int_0^{G(Lu,Mu,Nw)} f(t)\,dt\right\}$$

Consider ,

$$\leq\xi\left\{\int_0^{G(Pu,Qu,Rw)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Pu,Qu,Rw)} f(t)\,dt\right\}$$

$\therefore\ \xi\left\{\int_0^{G(u,u,Mu)} f(t)\,dt\right\}\leq\xi\left\{\int_0^{G(u,Mu,u)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(u,Mu,u)} f(t)\,dt\right\}$

i.e. $\xi\left\{\int_0^{G(u,u,Mu)} f(t)\,dt\right\}<\xi\left\{\int_0^{G(u,u,Mu)} f(t)\,dt\right\}$, which is a contradiction. $\therefore$ we get $Mu=u$

Hence $Mu=Qu=u$ i.e. u is fixed point of M and Q.

At last we prove that u is fixed point of N.

Consider , $\xi\left\{\int_0^{G(u,u,Nu)} f(t)\,dt\right\}=\xi\left\{\int_0^{G(Lu,Mu,Nu)} f(t)\,dt\right\}\leq\xi\left\{\int_0^{G(Pu,Qu,Ru)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Pu,Qu,Ru)} f(t)\,dt\right\}$

i.e. $\xi\left\{\int_0^{G(u,u,Nu)} f(t)\,dt\right\}\leq\xi\left\{\int_0^{G(u,u,Ru)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(u,u,Ru)} f(t)\,dt\right\}$

i.e. $\xi\left\{\int_0^{G(u,u,Nu)} f(t)\,dt\right\}<\xi\left\{\int_0^{G(u,u,Ru)} f(t)\,dt\right\}$, which means $\xi\left\{\int_0^{G(u,u,Nu)} f(t)\,dt\right\}<\xi\left\{\int_0^{G(u,u,Nu)} f(t)\,dt\right\}$ as $Nu=Ru$

Which implies that $Nu=u$. Hence we get $Nu=Ru=u$.

i.e. u is fixed point of N and R .

Thus u is the common fixed point of L,M,N,P,Q and R.

Now , we prove that u is the unique common fixed point of L,M,N,P,Q and R.

If possible , let us assume that μ is another fixed point of L,M,N,P,Q and R.

$$\therefore \quad \begin{aligned} &\xi\left\{\int_0^{G(u,u,\mu)} f(t)\,dt\right\} = \xi\left\{\int_0^{G(Lu,Mu,N\mu)} f(t)\,dt\right\} \le \\ &\xi\left\{\int_0^{G(Pu,Qu,R\mu)} f(t)\,dt\right\} - \eta\left\{\int_0^{G(Pu,Qu,R\mu)} f(t)\,dt\right\}\end{aligned}$$

$$= \xi\left\{\int_0^{G(u,u,\mu)} f(t)\,dt\right\} - \eta\left\{\int_0^{G(u,u,\mu)} f(t)\,dt\right\}$$

i.e. $\xi\left\{\int_0^{G(u,u,\mu)} f(t)\,dt\right\} < \xi\left\{\int_0^{G(u,u,\mu)} f(t)\,dt\right\}$, which is again a contradiction.

Hence finally we will have $u = \mu$.

Thus u is the unique common fixed point of L, M, N, P, Q and R.

Corollary 2.2: Let (X, G) be a complete G-metric space and $L, M, N, P : X \to X$ be mappings such that

i) $L(X) \subset P(X), M(X) \subset P(X)$,
$N(X) \subset P(X)$

ii) $$\xi\left\{\int_0^{G(Lx,My,Nz)} f(t)\,dt\right\} \le \xi\left\{\int_0^{G(Px,Py,Pz)} f(t)\,dt\right\} - \eta\left\{\int_0^{G(Px,Py,Pz)} f(t)\,dt\right\}$$

for all $x, y, z \in X$ where $\xi : [0,\infty) \to [0,\infty)$ is a continuous and non-decreasing function , $\eta : [0,\infty) \to [0,\infty)$ is a lower semi continuous and non-decreasing function such that $\xi(t) = \eta(t) = 0$ if and only if $t = 0$, also $f : [0,\infty) \to [0,\infty)$ is a Lebesgue integrable function which is summable on each compact subset of R^+ , non negative and such that for each $\in > 0$,

$$\int_0^{\in} f(t)\,dt > 0$$

iii) The pairs (L, P) , (M, P) , (N, P) are weakly compatible.

Then L, M, N, P have a unique common fixed point in X.

Proof : By taking $P = Q = R$ in Theorem 2.1 we get the proof.

Corollary 2.3: Let (X, G) be a complete G-metric space and $L, P : X \to X$ be mappings such that

i) $L(X) \subset P(X)$

ii) $$\xi\left\{\int_0^{G(Lx,Ly,Lz)} f(t)\,dt\right\} \le \xi\left\{\int_0^{G(Px,Py,Pz)} f(t)\,dt\right\} - \eta\left\{\int_0^{G(Px,Py,Pz)} f(t)\,dt\right\}$$

for all $x, y, z \in X$ where $\xi : [0,\infty) \to [0,\infty)$ is a continuous and non-decreasing function ,
$\eta : [0,\infty) \to [0,\infty)$ is a lower semi continuous and non-decreasing function such that
$\xi(t) = \eta(t) = 0$ if and only if $t = 0$, also

$f:[0,\infty)\to[0,\infty)$ is a Lebesgue integrable function

which is summable on each compact subset of R^+, non negative and such that for each

$\in>0$, $\int_0^{\in} f(t)\,dt>0$

iii) The pair (L,P) is weakly compatible.

Then L,P have a unique common fixed point in X.

Proof: By substituting $L = M = N$ and

$P = Q = R$ in Theorem 2.1 we get the proof.

Corollary 2.4: Let (X,G) be a complete G-metric space and $L,P:X\to X$ be mappings such that

i) $L(X)\subset P(X)$

ii) $G(Lx,Ly,Lz)\le G(Px,Py,Pz)$, for all

$x,y,z\in X$, $f:[0,\infty)\to[0,\infty)$ is a Lebesgue integrable function which is summable on each compact subset of R^+, non negative and such

that for each $\in>0$, $\int_0^{\in} f(t)\,dt>0$

iii) The pair (L,P) is weakly compatible.

Then L,P have a unique common fixed point in X.

Proof: By putting $f(t)=1$, $\xi(t)=t$ and $\eta(t)=0$ in Corollary 2.3, one can find the proof of Corollary 2.4

III. CONCLUSION

The Corollary 2.3 is similar to the result proved by Vishal Gupta and Naveen Mani [5] in complete metric space. So Main Result proved in this paper can be considered as a special case of earlier theorems already proved in complete metric space.

IV.REFERENCES

[1]. Alber Ya.I. & Guerre-Delabriere S. (1997). Principle of weakly contractive maps in Hilbert spaces, New Results in Operator theory and its applications in I.Gohberg and Y.Lyubich (Eds.), 98, Operator Theory: Advances and Applications,(7-22). Birkhauser, Basel, Switzerland.

[2]. Beg I. & Abbas M.,Coincidence point and invariant approximation for mappings satisfying generalized weak contractive condition. Fixed point theory and Appl., article ID 74503, 1-7.

[3]. Dutta P.N. & Choudhury B.S. (2008). A generalization of contraction principle in metric spaces. Fixed point theory and Appl., article ID406368, 1-8.

[4] . Fisher B., Common Fixed Point of Four Mappings, Bull. Inst. of .Math.Academia. Sinicia, 11(1983), 103-113.

[5] . Gupta V. & Mani N. , A Common Fixed Point Theorem for Two Weakly Compatible Mappings Satisfying a New Contractive Condition of Integral Type, Mathematical Theory and Modeling , Vol.1, No.1, 2011

[6] . Jungck G. (1976). Commuting mappings and fixed points. Amer.Math.Monthly, 83, 261-263.

[7] . Jungck G. (1986). Compatible mappings and common fixed points. Internat. J. Math. Sci ., 9, 43-49.

[8] . Jungck, G. & Rhoades, B.E. (1998). Fixed points for set valued functions without continuity. Indian J.Pure. Appl.Math., 29, No. 3, 227-238.

[9] . Khan M.S., Swaleh M. & Sessa S. (1984). Fixed point theorems by altering distances between the points, Bull. Austral. Math. Soc., 30, 1-9.

[10] . Mustafa Z., Sims B., A new approach to generalized metric spaces, J.Nonlinear Convex Anal. 7 (2006), 289-297.

[11] . Pant R.P. (1994). Common fixed points of non commuting mappings. J.Math.Anal. Appl., 188, 436-440.

[12] . Rhoades B.E. (2001). Some theorems on weakly contractive maps, Nonlinear Analysis: Theory. Methods&Applications,47 (4), 2683-2693.

[13] . Sastry K.P.R., Naidu S.V.R., Babu, G.V.R. & Naidu, G.A. (2000). Generalizations of common fixed point theorems for weakly commuting mappings by altering distances. Tamkang J.ath., 31, 243-250.

[14] . Sessa S. (1982). On weak commutative condition of mappings in fixed point considerations. Publ. Inst. Math N.S., 32, no.46, 149-153.

[15] . Suzuki T. (2008). A generalized Banach contraction principle that characterizes metric completeness. Amer.Math.soc.,136 (5), 1861-1869.

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Some Generalization of Certain Commutativity Theorems on Semi-Prime Rings

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ABSTRACT

In this paper we have proved that, let $n > 1$ be a fixed positive integer and R be a semi-prime ring which satisfies any one of the following polynomial identities

[A] (i) $[[(xy)^n - x^n y^n, x], z] = 0$ or (ii) $[[(xy)^n - y^n x^n, x], z] = 0$

[B] (i) $[[(xy)^n - x^n y^n, y], z] = 0$ or (ii) $[[(xy)^n - y^n x^n, y], z] = 0$ for all $x, y, z \in R$, then R is commutative.

Keywords: $[x, y] = xy - yx$, semi-prime ring, semi-simple rings, nilpotent, monomials

I. INTRODUCTION

If a ring R, satisfies the identity $(xy)^n = x^n y^n$, for all $x, y \in R$ and $n > 1$ a fixed positive integer then Ram Awtar [2] and Abu-Khuzam [4] have shown R to commutative, imposing torsion condition on the additive group R^+.

Later Abu-Khuzam [5], proved " If R is semi-prime ring in which, for x in all, there exists an integer $n = n(x) > 1$ such that $(xy)^n = x^n y^n$, for all $x, y \in R$, then R is commutative. Quadri and Ashraf [8] generalized Abu-Khuzam's [5] result by taking $[(xy)^n - x^n y^n, z] = 0$ or $[(xy)^n - y^n x^n, z] = 0$ for all $x, y, z \in R$ and $n > 1$ a fixed positive integer, to infer the ring to be commutative.

We further generalized the result of Quadri and Ashraf [8] as follows

that " Let $n > 1$ be a fixed positive integer and R be a semi-prime ring which satisfies any of the following polynomial identities

[A] (i) $[[(xy)^n - x^n y^n, x], z] = 0$ or (ii) $[[(xy)^n - y^n x^n, x], z] = 0$

[B] (i) $[[(xy)^n - x^n y^n, y], z] = 0$ or (ii) $[[(xy)^n - y^n x^n, y], z] = 0$ for all $x, y, z \in R$, then R is commutative.

II. PRELIMINARIES LEMMAS

In this section, we mention some Preliminaries Lemmas which are required to prove preparatory lemmas, mention in next section.

Let R be associative ring and f a set of polynomials (each in some finite number of non commuting indeterminates) with integer coefficient. R will be called an f - rings if and only if for every finite sub set S of R there is an **f** in f which vanishes identically on S. Thus certain kinds of rings can be characterized as f rings for appropriate choice of f: nil rings, with f the set of polynomials x^n for all positive integer n; locally nilpotent rings, with f the set of monomials $x_1x_2x_3 \dots x_n$ in n indeterminates for all n; nilpotent rings, with f the single monomial $x_1x_2x_3 \dots x_n$ for some n; rings satisfying the polynomial identity, with f finite (or equivalently with f containing a single polynomial) and so forth.

Now we mention without pro0f an important theorem of T.P. Kezlan [7] as under

2.1. Lemma- T.P. Kezlan [7, Theorem 5]

Suppose that f is finite. Then the following are equivalent:

(i) Every f -ring has nil commutator ideal.

(ii) For every prime p the ring GF$(p)_2$ of 2×2 matrices over GF(p) is not an f -ring.

Next we state a result of bell [3]

2.2. Lemma: Bell [3, Theorem 1]

Let R be a ring satisfying an *identity* $q(x) = 0$, Where q(x) is a polynomial in a finite number of non-commuting indeterminates, its coefficient being integers with the highest common factor 1. If there exists no prime p for which the ring of 2×2 matrices over $GF(p)$ satisfies $q(x) = 0$, then R has nil commutator ideal and the nilpotent elements of R form an ideal.

Proof:

The argument in the proof of lemma 2.1 shows that semi-simple rings satisfying the given identity are commutative. Now let R be any ring satisfying $q(x) = 0$ and p an arbitrary proper prime ideal. Then the prime ring $\bar{R} = \frac{R}{p}$ satisfies the same identity and by a result of Amitsur [1] can be imbedded in a ring S which is simple with 1 and also satisfies $q(x) = 0$, hence is commutative. It follows that if $x \in R$ and $u, v \in R$ are such that $u^n = v^n = 0$, then their canonical images in $\bar{R}$ satisfy $(\bar{u} + \bar{v})^{m+n+1} = (\bar{u}\bar{x})^n = (\bar{x}\bar{u})^n = 0$. both conclusions of our theorem. Now follow from the fact that the intersection of the prime ideals of R is nil ideal.

After this we quote a lemma of Herstein [6] without proof as follows:

2.3. Lemma: Herstein [6, lemma 2.1.1]

If R is any ring and $p \neq 0$ is a nil right ideal then if p satisfies a polynomial identity, R has a non-zero nilpotent ideal. In particular if every $x \in p$ satisfies $x^n = 0$ where n is a fixed integer then R cannot be semi-prime.

III. PREPARATORY RESULT

In this section we prove a Preparatory lemma which assists the proof our main theorem.

3.1. Lemma:

Let R be a semi-prime ring satisfying any of the following polynomial identities

[A] (i) $[[(xy)^n - x^n y^n, x], z] = 0$ or (ii) $[[(xy)^n - y^n x^n, x], z] = 0$

[B] (i) $[[(xy)^n - x^n y^n, y], z] = 0$ or (ii) $[[(xy)^n - y^n x^n, y], z] = 0$ for all $x, y, z \in R$, and $n > 1$ be a fixed positive integer then R contains no non-zero nilpotent elements.

Proof [A] (i)

Let $x \neq 0$ be an element of such that $x^2 = 0$. Hypothesis

[A] (i) $[[(xy)^n - x^n y^n, x], z] = 0 => [\{(xy)^n - x^n y^n\}x - x\{(xy)^n - x^n y^n\}, z] = 0$

Using $x^2 = 0$, the above equation yields

$[(xy)^n x, z] = 0$ for all $y, z \in R$.

Now replace z by y in the equation, to obtain

$[(xy)^n x, y] = 0$

$=> [(xy)^n xy - y(xy)^n x] = 0$

$=> [(xy)^{n+1} - y(xy)^n x] = 0$

$=> (xy)^{n+1} y\{(xy)(xy) \ldots (n) times \ldots (xy)\}x = 0$

$=> (xy)^{n+1}\{(yx)(yx) \ldots (n+1) times \ldots (yx)\} = 0$

$=> (xy)^{n+1} - (yx)^{n+1} = 0$ --- (1)

Multiply with xy on the RHS in equation (1) and use $x^2 = 0$, to get

$(xy)^{n+2} = 0$ For all $y \in R$ --- (2) $=> xR = 0$, because if $xR \neq 0$, then the equation (2) asserts that xR is non-zero nil right ideal satisfying the identity $(t)^{n+2} = 0$ for all $t \in xR$. But by lemma 2.3 R has a non-zero nilpotent ideal which is contradiction, since R is semi-prime. Thus, $xR = 0$, and hence $xRx = 0$. This implies that $x = 0$ as R is semi-prime, again contradiction to $x \neq 0$

On the similar lines we can prove for the polynomial identities [A] (ii), [B] (i) and [B] (ii)

IV. MAIN RESULT

In this section, we prove our main theorem and gives a counter example to show that main theorem does not hold for arbitrary rings.

4.1 Theorem:

In this paper we have proved that "let $n > 1$ be a fixed positive integer and R be a semi-prime ring satisfying the polynomial identities of lemma 3.1, then R is commutative.

Proof:

We shall prove the result for rings satisfying the identity $[[(xy)^n - x^n y^n, x], z] = 0$. For other cases, we can get the result by proceeding on the similar lines.

If R is semi-prime ring satisfying the identity

$q(x, y.z) = [\{(xy)^n - x^n y^n\}x - x\{(xy)^n - x^n y^n\}, z] = 0$

$=> [\{(xy)^n - x^n y^n\}x - x\{(xy)^n - x^n y^n\}]z - z[\{(xy)^n - x^n y^n\}x - x\{(xy)^n - x^n y^n\}] = 0$

$=> (xy)^n xz - x^n y^n xz - x(xy)^n z + x^{n+1} y^n z - z(xy)^n x + zx^n y^n x + z x\{(xy)^n - zx^{n+1} y^n = 0$

Then it is isomorphic to a sub direct sum of prime ring R_α each of which as a homomorphic image of R satisfies the hypothesis placed on R. Hence we can assume that R is prime ring satisfying

$q(x, y.z) = (xy)^n xz - x^n y^n xz - x(xy)^n z + x^{n+1} y^n z - z(xy)^n x + zx^n y^n x + z x\{(xy)^n - zx^{n+1} y^n = 0$

which is polynomial identity with co-prime integral coefficients.

Now if we consider $x = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $y = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, $and\ z = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, we find that no 2×2 matrix ring over $GF(p)$, p a prime, satisfies the forgoing identity, For example choose $n = 2$ and check the given identity for chosen $x\ and\ y$.

$(xy) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$(xy)^2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$x^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$y^2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$x^2y^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Putting the above values in the LHS of given identity

$[[(xy)^2 - x^2y^2, x], z]]$

$=[[\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}], \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}]$

$=[[\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}], \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}]$

$= [\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}]$

$=[\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}]$

$=[\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}]$

$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$[[(xy)^2 - x^2y^2, x], z] \neq 0$

Thus we have shown that no 2×2 matrix ring over $GF(p)$, p a prime, satisfies the identity $[[(xy)^n - x^ny^n, x], z] = 0$, where $n > 1$.

Hence by lemma 2.2, R has a nil commutator ideal. But by lemma 3.1, R has no non-zero nilpotent elements. Thus the commutator ideal is zero and R is commutative.

4.1 Counter Example

The following counter example shows that the above main theorem is not true for arbitrary ring.

Let D be division ring and D_k be the complete matrix ring of all $k \times k$ matrices over D where $k > 2$.

Let $A_k = \{(a_{ij}) \in D_k \,/\, a_{ij} = 0\ (i > j)\}$

Then A_3 is non commutative nilpotent ring of index 3, which is not a semi-prime ring. However , it satisfies the identities given in the main theorem.

Let $x = \begin{bmatrix} 0 & a_1 & b_1 \\ 0 & 0 & c_1 \\ 0 & 0 & 0 \end{bmatrix}$, $y = \begin{bmatrix} 0 & a_2 & b_2 \\ 0 & 0 & c_2 \\ 0 & 0 & 0 \end{bmatrix}$, $z = \begin{bmatrix} 0 & a_3 & b_3 \\ 0 & 0 & c_3 \\ 0 & 0 & 0 \end{bmatrix}$ be elements of A_3 , $where\ a_{ij} \in D$.

First we show that A_3 is non commutative. Clearly

$$xy = \begin{bmatrix} 0 & a_1 & b_1 \\ 0 & 0 & c_1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & a_2 & b_2 \\ 0 & 0 & c_2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & a_1\,c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \; and \; yx = \begin{bmatrix} 0 & a_2 & b_2 \\ 0 & 0 & c_2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & a_1 & b_1 \\ 0 & 0 & c_1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & a_2\,c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since $a_1\,c_2 \neq a_2\,c_1$, because these elements are in D, Hence, A_3 is non-commutative, i.e. $xy \neq yx$.

Now we show that A_3 is nilpotent ring of index 3. Note that each element of A_3^3 is of the form xyz and it is equal to zero. Hence $A_3^3 = 0$.

We check that $xyz = 0$

$$\text{L.H.S.} = xy.z = \begin{bmatrix} 0 & 0 & a_1\,c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & a_3 & b_3 \\ 0 & 0 & c_3 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This also shows $xRx = 0$ though $x \neq 0$.

Hence A_3 is non-commutative nilpotent ring of index 3, and it is not semi-prime ring.

However, A_3 satisfies all the identities given in the main theorem. For this, we choose n=2, in the identity $[[(\;xy)^n - x^n y^n, x], z] = 0$, to obtain $[[(\;xy)^2 - x^2 y^2, x], z] = 0$. Clearly $(xy)(xy) =$

$$\begin{bmatrix} 0 & 0 & a_1\,c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & a_1\,c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x^2 = \begin{bmatrix} 0 & a_1 & b_1 \\ 0 & 0 & c_1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & a_1 & b_1 \\ 0 & 0 & c_1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & a_1\,c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, y^2 \begin{bmatrix} 0 & a_2 & b_2 \\ 0 & 0 & c_2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & a_2 & b_2 \\ 0 & 0 & c_2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & a_2\,c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x^2y^2 = \begin{bmatrix} 0 & 0 & a_1\,c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & a_2\,c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, Cosequenty \;\; (xy)^2 - x^2y^2 \; = 0.$$

This finally yields $[[\,0, x], z] = [0, z] = 0$

Similarly we can check other identities also.

V. REFERENCES

[1] . Amitsur.S.A : “Prime-rings having polynomial identities with arbitrary coefficients” Proc. London Math. Soc. (3) (1967), 470-486

[2] . Awtar.R. : “On the commutativity of non-associative rings.” Publ.Math. Debrecen, 22 (1975) 177-188

[3] . Bell.H.E : “On some commutativity theorem of Herstin.” Arch.Math.(Basel) 24 (1973) 34-38.

[4] . Hazar abu-khuzam : “A commutativity theorem for rings.”Math.Japonica 25 (1980) 593-595.

[5] . Hazar abu-khuzam : “A commutativity theorem for semi-prime rings.” Bull. Astral. Math. Soc. 27 (1983), 221-224.

[6] . Herstien,I.N : “Rings with involution.” University of Chicago and Londan 1964.

[7] . Kezlan,T.P : “Rings in which certain subsets satisfy polynomial identities.” Trans.Amer. Math. Soc. 125 (1966) 414-421.

[8] . Quadri.M.A and Mohd. Ashraf : “A theorem on commutativity of semi-prime rings” Bull. Astral. Math. Soc.14 (1986) 411-413

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Analysis of Laplace Transform & Its Specific Applications in Engineering

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ABSTRACT

Entering in the late 20th century after being popularized by a famous Electrical Engineer, knowledge on how to do the Laplace Transform has become a necessity for many fields. Anyone having background of Mathematics including Engineering & Sciences are always exposed to Differential Equations, Laplace transform etc. Through this paper, we have tried to bring out some specific Applications of Laplace Transform in different Engineering fields specially solving Differential equations by using Laplace Transform in turn showcasing more important uses of the transform.

Keywords : Laplace Transform, Differential Equation, Inverse Laplace Transform.

I. INTRODUCTION

Laplace transform, in Mathematics, a particular integral transform invented by the French mathematician Pierre-Simon Laplace (1749–1827), and systematically developed by the British physicist Oliver Heaviside (1850–1925), to simplify the solution of many Differential Equations that describe physical processes. A Laplace transform is an extremely diverse function that can transform a real function of time t to one in the complex plane s, referred to as the frequency domain.

Laplace transform makes it easier to solve the increasing complexity of engineering problems and has wide applications of Laplace in Electric circuit analysis, Communication Engineering, Nuclear Physics as well as Automation Engineering, Control Engineering and Signal processing, Probability theory, determination of transfer function in Mechanical system and many more. The ready tables of Laplace Transforms reduce the problems of solving differential equations to mere algebraic manipulation.

II. DEFINITION OF INTEGRAL TRANSFORM

Let *K(s, t)* be a function of two variables *s* and *t* where *s* is a parameter [$s \in$ R or C] independent of t. Then the function *F(s)* defined by an Integral which is convergent. i.e.,

$F(s) = \int_{-\infty}^{\infty} K(s,t)\ f(t)\ dt$ is called the Integral Transform of the function f(t) and is denoted by I{f(t)} where K(s, t) is kernel of the transformation.

A. Definition of Laplace Transform

If kernel *K(s, t)* is defined as $K(s,t) = \begin{cases} 0, & \forall\ t < 0\ \ or\ \ \frac{1}{t} > 0 \\ e^{-st} & \forall\ t \geq 0 \end{cases}$ then

$F(s) = \int_{0}^{\infty} e^{-st}\ f(t)\ dt$ is called "Laplace Transform" of the function f(t) and is also denoted by L{f(t)} or $\overline{f}(s)$.

$$\therefore L[f(t)]\ or\ \overline{f}(s)\ \ or\ F(s) = \int_{0}^{\infty} e^{-st}\ f(t)\ dt$$

B. Inverse Laplace Transform

If L {f (t) } = $\overline{f}(s) \Rightarrow$ f (t) = L^{-1} { $\overline{f}(s)$ }. Then f(t) is called the inverse Laplace Transform of $\overline{f}(s)$. Here operator L which transforms f(t) into $\overline{f}(s)$ is called "The Laplace Transforms Operator".

C. Piecewise or sectionally continuous

A function f (t) is said to be piecewise or sectionally continuous on a closed interval $a \leq t \leq b$, if it is defined on that interval can be divided into finite number of sub-intervals in each of which f(t) is continuous and has finite left limit and right hand limits.

$\underset{t \to 0^-}{\text{Lim}}\ f(t) = \underset{t \to 0^+}{\text{Lim}}\ f(t) = \underset{t \to 0}{\text{Lim}}\ f(t) = \text{finite f}$ say

$\forall\ a \leq t \leq b$, therefore, f is continuous.

D. Functions of an Exponential order

A function f(t) is said to be an exponential order α as t tends to ∞ if there exist a positive real number M a number α and a finite number t_0 such that $| f(t) | < M e^{\alpha t}$ **or** $| e^{-\alpha t} f(t) | < M, \forall\ t \geq t_0$.

Note: If a function f(t) is of an exponential order α. It is also of β such that $\beta > \alpha$.

III. LAPLACE TRANSFORM OF SOME SPECIAL FUNCTIONS

A. Periodic function

A function f (t) is said to be a periodic function of period T > 0 if

f(t) = f(T + t) = f(2T + t) = f(3T + t) = ... = f(nT + t).

sin t, cos t are periodic functions of period 2π.

The Laplace transform of a piecewise periodic function f (t) with period T is

$$L\{ f(t) \} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} . f(t)\, dt \; ; s > 0$$

B. Unit step function or Heaviside Unit function

In engineering, many times we come across such fractions of which inverse Laplace is either very difficult or cannot be obtained by the known formulae. To overcome such problem, Unit step function (Heaviside's Unit Function) has been introduced. The unit step function is defined as follows

$$u(t-a) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t \geq a \end{cases}$$

Where a is always positive.

As a particular case when a = 0 then

$$u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

Also,

$L[u(t-a)] = \int_0^\infty e^{-st} u(t-a) dt = \int_0^a e^{-st}(0)\, dt + \int_a^\infty e^{-st}(1)\, dt$ $L[u(t-a)] = \left. \frac{e^{-st}}{-s} \right|_a^\infty = \frac{e^{-as}}{s}$ and in particular when $a = 0$, $L[u(t)] = \frac{1}{s}$

Generally the unit step function in mechanical engineering comes into picture as a force suddenly applied to a machine or a machine component, where as in electrical engineering it manifests as an electromotive force of a battery in circuit.

C. Dirac's Delta function or Unit impulse function

A force of very high magnitude applied for an instant producing a large effect is called impulse .The function representing the impulse is called dirac-Delta function.

More precisely, Dirac-Delta function is the limiting form of the function $\delta_\varepsilon(t-a) = \begin{cases} \frac{1}{\varepsilon} & , a \leq t \leq a+\varepsilon \\ 0 & , \text{otherwise} \end{cases}$ as $\varepsilon \to 0$.This fact can be further represented as $\delta(t-a) = \begin{cases} \infty & , t = a \\ 0 & , t \neq a \end{cases}$ such that $\int_0^\infty \delta(t-a)dt = 1 \;\; for \;\; a \geq 0$

The Dirac-Delta function is mainly originated from the concept of strongly peaked or concentrated functions. In electrical circuits strongly peaked currents of extremely short duration occurs frequently in switching processes. In mechanics, the impulse of the blow is equal to momentum when a body is set in motion from rest by a sudden -blow.

$$L[\delta(t-a)] = \int_0^{\infty} e^{-st}\delta(t-a)dt$$

$$= \int_0^{a} e^{-st}(0)\,dt + \int_a^{a+\varepsilon} e^{-st}(\frac{1}{\varepsilon})\,dt + \int_{a+\varepsilon}^{\infty} e^{-st}(0)\,dt$$

$$= \frac{1}{\varepsilon}\left[\frac{e^{-st}}{-s}\right]_a^{a+\varepsilon} = \frac{e^{-as}}{s}\left[\frac{1-e^{-s\varepsilon}}{\varepsilon}\right]$$

Taking limit as $\varepsilon \to 0$

$$L[\delta(t-a)] = \lim_{\varepsilon\to 0}\frac{e^{-as}}{s}\left[\frac{1-e^{-s\varepsilon}}{\varepsilon}\right] = e^{-as}$$

IV. APPLICATIONS

A. In Frequency Response system

An important property of Laplace Transform systems is that if the input given to the system is sinusoidal, then the output must be sinusoidal at the same frequency but with different magnitude and phase. These differences as a function of frequency are known as the frequency response of the system. Frequency-domain methods are often used for analyzing LTI single-input/single output (SISO) systems.

B. Digital Signal Processing

It's use is observed while sending signals over two-way communication medium (for ex. FM/AM stereos, 2 way radio sets, cellular phones.) where medium wave's time functions are converted to frequency functions.

C. Traffic Engineering

It is widely implemented for quantifying speed control through modelling of road bumps by converting to hollow rectangular shape. In doing so, the vehicle is considered as the classical one-degree-of-freedom system whose base follows the road profile, thus approximated by Laplace Transform.

D. Medical Field

One can corelate blood-velocity/time wave form over cardiac cycle from common femoral artery as an approximation of Laplace Transform.

E. Electric circuits

Ex. Use Laplace transform method to obtain the charge at any instant of a capacitor which is discharged in R–C–L circuit, after the switch is closed if R = 2.25 ohms, self-inductance L = 1 Henry, capacitance, C = 2 farads, and the capacitor has initially a charge of 100 coulombs. Initially the switch is open and therefore, no current is flowing.

Solution: The L-C-R circuit equation is

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = \frac{E}{L}$$

On Substituting values, we get

$$\frac{d^2q}{dt^2} + \frac{9}{4}\frac{dq}{dt} + \frac{q}{2} = 0$$

Taking Laplace transform on both the side, we get

$$\left[s^2+\frac{9}{4}s+\frac{1}{2}\right]\bar{q}(s)-100s-225=0$$

$$\bar{q}(s)=\frac{100(4s+9)}{(4s^2+9s+2)}=\frac{100}{7}\left[\frac{32}{4s+1}-\frac{1}{s+2}\right]$$

$$\bar{q}(s)=\frac{100}{7}\left[\frac{8}{s+\frac{1}{4}}-\frac{1}{s+2}\right]$$

Taking inverse Laplace transform on both sides, we get $q(t)=\frac{100}{7}\left[8e^{-\frac{t}{4}}-e^{-2t}\right]$

F. Vibration Mechanics

Ex. Obtain the equation for the forced oscillations of mass *m* attached to the lower end of an elastic spring whose upper end is fixed and whose stiffness is *k*, when the driving force is F_0 *sinat*. Solve this equation (Using the Laplace Transforms) when $a^2 \neq k/m$, given that initially velocity and displacement (from equilibrium position) are zero.

Solution: This problem deals with forced oscillations (without damping)

Let the periodic force be F_0 *sinat* then equation of motion becomes

$$m\,\frac{d^2x}{dt^2}=mg-k\,(e+x)+\;F_0\;sinat$$

where, x is length of stretched portion of the spring (displacement) after time t,

e is the elongation produced by the mass m,

k is the restoring force per unit stretch of the spring due to elasticity;

a is any arbitrary constant and p is any scalar.

Since tension $mg = k\,e$, equation becomes

$$\frac{d^2x}{dt^2}+\;n^2x\;=\;\frac{F_0}{m}\;sinat$$

Taking L.T. on both sides and using $x(0) = 0,\ x'(0) = 0$

$$\overline{x(s)}\;=\;\frac{a\,F_0}{m}\,\frac{1}{(s^2+a^2)\,(s^2+n^2)}\quad \mu\;=\;\sqrt{\frac{k}{m}}$$

Resolving by Partial fractions we have A = 0, B = $\frac{1}{n^2-a^2}$, C = 0, D = $\frac{-1}{n^2-a^2}$

Thus, we get $\overline{x(s)}\;=\;\frac{a\,F_0}{m\,(n^2-a^2)}\left[\frac{1}{(s^2+a^2)}-\frac{1}{(s^2+n^2)}\right]$

Taking inverse Laplace Transform

$x(t)\;=\;\frac{F_0}{m\;n\;(n^2-a^2)}\;[n\;sinat\;-\;a\;sinnt]$, where $n\;=\;\sqrt{\frac{k}{m}},\frac{k}{m}\neq\;a^2$ is the required solution.

V. CONCLUSIONS

Thus, from the above analysis of definition & its forms as well as illustrious various applications of Laplace transform one can infer that it is an indispensable tool in solving linear ordinary and partial differential equations with constant coefficients under suitable initial and boundary conditions by first finding the general solution and then evaluating from it the arbitrary constants. Modern society would have witnessed set back in rapid increase of technology and growth might have stopped if such varied and diverse applications are not brought forward. For partial differential equations involving two independent variables, Laplace transform is applied to one of the variables and the resulting differential equation in the second variable is solved by the usual method of ordinary differential equations. Hopefully, the applications discussed of Laplace transform in this paper will prove beneficial to students and researchers.

VI.REFERENCES

[1] . B.V. Ramanna, Higher Engineering Mathematics, Tata Mc Graw Hill Publication.
[2] . H.K. Dass and Er. Rajnish Verma, Higher Engineering Mathematics, S. Chand Publication, New Delhi.
[3] . Sarina Adhikari, Laplace Transform and its applications, University of Tennessee. (Department of Electrical Engineering).
[4] . Ananda K. and Gangadharaiah Y. H, Applications of Laplace Transforms in Engineering and Economics, IJTRD, 2016
[5] . Prof. L.S.Sawant, Applications of Laplace transform in Engineering Fields, International Research Journal of Engineering and Technology (IRJET), e-ISSN: 2395-0056 Volume: 05 Issue: 05 | May-2018 ISSN: 2395-0072.

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Application of Game Theory Model using Regression for the Graph Analytics Parameters of the Social Networking

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ABSTRACT

The research on social network is constantly changing due to the variety of social networks and the abundance in social network data. For mathematical and statistical modeling, this change brings about a fair share of opportunities as well as challenges.

In statistical modeling, Regression analysis is a method that identifies the effect of an explanatory variable on the dependent variable or predicts the value of dependent variable for entities wherein some information on the explanatory variables is known. In order to determine payoff in a two-person game theory model, various parameters of the Regression model Y on X and X on Y, can be further utilized. This research provides application evidence for the two giants of social network platform viz. Twitter and Facebook. The different parameters of graph analytics used by these two social media platforms form the basis of strategies used in this research. In order to get an idea for the complete process and receive outcomes for taking proper decision for two players, algorithm and a case study can be performed. In this study, the result of the game theory model gives the two players an opportunity to optimize their strategies with respect to graph analytics. The proposed algorithm can be extended to a larger number of players. This provides a substantial scalability for the innovative competitor analysis tool.

Keywords: Regression Analysis, Game Theory, Social Network, Graph Analytics.

I.INTRODUCTION

The ‘social network’ comprises of a set of players and a set of relation between those players. A social network is a social structure consisting of a set of social players (such as individuals or organizations), multiple dyadic tie sets and other social interactions between players. Social network perspective provides a set of methods to analyze the structure of social entities along with a variety of models that describe the patterns observed in these structures. With the use of social network analysis, these structures are studied to find significant units, recognize the universal as well as local patterns and scrutinize the network aspects. An integral interdisciplinary academic field, social network and its analysis arose from diverse fields such as social psychology, sociology, statistics, and graph theory.

A social networking site is a platform to build social networks or social relations between people who share common interests, real-life connections, backgrounds or activities. From several years, mathematical analysis of network is fruitfully applied to cater to this system., The basic assumptions in the networks analysis have transformed considerably, owing to advancement on the internet and social networks.

The mathematical sciences have remarkably contributed to this field with its emphasis on the progress of random graph models that represent few of the qualitative properties observed of wide range network data. With the use of recently developed mathematical models, several features of social network can be understood with no difficulties. Degree of connectivity is a feature wherein far off parts of a population are connected vide short paths. This is one feature of the network that brings the world closer. The short paths can be easily looked out for with the use of decentralized search algorithms.

Game theory can be defined as the process of modeling the strategic interaction between two or more players in a situation containing set rules and outcomes. The basic terms linked to a game theory model are as shown in the following figure-

Figure 1: Key terms of Game theory Model

Players: A strategic decision-maker within the context of a game.

Rules: In game theory, there are multiple players, actions, payoffs and information. It is referred as the rule of the game wherein try to maximize their payoffs.

Payoff: The value linked with the probable outcome of a game.

The basic purpose of the regression model is to provide a mathematical relationship between the two set variables where one is dependent and the other is independent. The linear regression equation Y on X is given by- Y =ax+b, where Y, is called as dependent variable, X is a independent variable, a is called as slope and b is the intercept value.

Similarly, the linear regression equation X on Y is given by-

X =cY+d, here now X is called as dependent variable, Y is an independent variable, c is called as slope and d is the intercept value.

One of the properties of regression analysis is that the lines of regression pass through their mean values.
In order to develop the innovative competitor analysis tool based on the concept of game theory model, all these mathematical foundations are applied wherein, the pay-off values are arrived on by processing the regression analysis concept.

II.LITERATURE SURVEY

This section is split into two sub-sections. In the first section, context of competitor analysis, game theory model and regression analysis are discussed. In the later section, availability of literature on social networking Platform viz. Twitter and Facebook. These above-mentioned sections aid in finding the strategies for the case study.

With the help of text mining, Wo He and et. al. (2013) operated on the notion of social media competitive analysis. Using the customer information, strategies of three pizza companies were analyzed [1]. Alex Yaw Adom and et.al (2016) analyzed the types of competitors as well as the spirit of competition. His study tells how the competitor analysis helps to take decisions and find new opportunities as opposed to the numerous competitors [2]. N. V. Vaidya, and N. N. Kasturiwale (2014) develop a new approach to solve the game theory model. This could provide a faster approach as compared to the traditional methods for solving the game theory model [3]. Satyajit S. Uparkar and et. al. (2019) worked on analytical study of the application of the Game theory model on social networking players. The payoff matrix of their game theory model was constructed based on regression model approach [4]. Sajedeh Norozpour and Mehdi Safaei (2020) represented the evidence of real-world events in terms of mathematical identities using the game theory model. The application provides the key terms and their utility associated with the Game theory concept. The major finding includes the Game theory model is a decision-making tool where the companies can optimize their business strategies as compared with their business competitors [5]. Anuja Arora and et.al (2020) used social media post for defining the competitive analysis. They have used the concept of logistic regression analysis for the analysis and interpretation of the results [6].

T Spiliotopoulos, Oakley has carried out research to identify how people sail through the social media ecosystem and choose the social network site (SNS) they would like to use. Thus, the current study uses behavioral data to observe the findings of technology non-use and takes inferences from the 'Uses and gratifications (U&G) theory' in order to compare the motives for the use of Face-book and Twitter [7]. Jayakumar Subramanian Aditya Mahajan Aditya A. Paranjape have studies the determination of leader-follower system's structural controllability that is defined over directed graphs. The notions of graph structural controllability and strong graph structural controllability in leader-follower systems on directed graphs are then identified. This is followed by proving that for a graph structural controllability over directed graphs, accessibility is an essential and adequate condition. This condition is subsequently identified which is followed by deriving a sufficient condition for controllability wherein two graphs are cascaded. The examples that demonstrate these ideas are then presented. [8]. Thus, the concept of Leader-follower is added as a strategy in our work. Through the method of searching for a keyword over the attributed graph, Sanket Chobe and Justin Zhan have proposed an algorithm for online community recognition. By means of keyword search method through this algorithm, the community detection can be further advanced. This allows online restoration of personalized and widespread communities [9]. Therefore, Community Detection is added as third variable for our research work. Manishka Gautam, Himanshu Sharma, Mayank Chauhan, Himanshu Shukla, Nidhi Tawra have proposed a design that offers a new recommendation system for alumni association. Based on their respective whereabouts and abilities, this system suggests 'friends' to the users. The project intents to connect the students, alumni and faculty through a single network where they can thus connect and collaborate for further ventures. To cater to the need of career advancement, the system provides the platform for students. The intent of project is to provide a platform to the college authority which along with assisting in the demonstration of college activities, is also a full-fledged social network that includes ideas based on graph database [10]. Thus, the concept of Friend-of-friend is added as a fourth variable. By using the concept of Graph database, Satyajit S. Uparkar and et.al. (2016) offered ways of identifying the influencers in social Networking domain. This study is based on the collection of twitter data. This same concept can be extended to the collection of Facebook data as well. Thus, the first attribute to strategy building i.e., influencers can be provided [11]. Shalin Hai-Jew (2020) has contributed a concept of whistleblowing on the Transnational Social Media: From Micro-to-Mass Scales, Privately and Publicly. The graph provided for Whistleblower Article-Article Network on Wikipedia of degree 1 and Whistleblower" Related Tags Network on Flickr, reveal add a critical factor to this study. Thus, the concept of the Whistle blower is added to the corpus of twitter and Facebook [12].

Innovative algorithm:

For developing an innovative competitor analysis tool initially for two players is as follows-

1. Start
2. Identifying the two players of game theory.
3. Identify the strategies to be optimized for decision making, common to both the players.
4. Define the target, scope, sample size for data collection.
5. Prepare the questionnaire for data collection.
1. 6. Collect the data for pilot survey to determine the reliability of model consideration.
6. Reliability tool is passed.
   a. Yes: Go to next step.
   b. No: Perform modification and go to step 4
7. Perform Explanatory statistics/ Descriptive statistics to identify the initial trends for two players.
8. Perform the steps for Competitive analysis subroutine.
9. Interpret the results for optimal strategies in each case
10. Stop

The steps for Competitive analysis subroutine

1. Start
2. Extract the dataset for player A and B
3. For each factor of A, calculate linear regression equation A and B and mean values of Bs
4. Store slope, intercept and mean values of Bs.

5. For each values of B, calculate linear regression equation B and A and mean values of A's
6. Store slope, intercept and mean values of As.
7. Is information share between A and B

   NO: Calculate payoff values of A and B using intercept values of B and Calculate payoff values of B and A using intercept values of A.

   YES: Calculate payoff values of A and B using intercept, slope and mean values of B and Calculate payoff values of B and A using intercept slope and mean values of A.
8. Build the required payoff matrices for all the cases in tuple format of (major, minor)
9. Solve the Games for the values of the game and the optimal strategies in each case.
10. Return to main algorithm for the final interpretation.

Case Study based on Graph Analytical Parameters:

To check the feasibility of the innovative algorithm, two giant players of Social Networking Platform – Twitter and Facebook were identified for their common strategies based on Graph analytics. The execution of the mentioned algorithm for these two players is as follows-

- Game Theory model:

  Five Identified factors for these two players are-
  - Leader and Followers
  - Community Detection
  - Friend of Friend
  - Influencers
  - Whistle blowers

Sample Size and Scope:

As per statistics of first Quarter of 2021, In India, there are around 8-12 lacs IT employees' workings as data scientise/social media scientise. These categories of IT professional work on social networking analytics, algorithms, trends etc. So, the sample size of 385 was decided for this study at 5% level of significance. For this alumnus of various college were contact for the references.

Data Collection approach:

Online Google Forms was provided to the social media professional. The questionnaire containing two section one for demographic information and another of the five identified strategies on 10-point scale was framed.

Reliability Study:

Cronbach alpha test on Pilot Survey for first 50 samples. Any value of alpha greater than 0.7 reflects the acceptance of the questionnaires. The reading obtained from SPSS indicates that the questionnaire designed for all the factors seems self-sufficient to proceed with the remaining responses.

Exploratory Statistics:

The demographic variables like data/ social media analytic scientist worked on these platforms and the distribution of their work experience are given below:

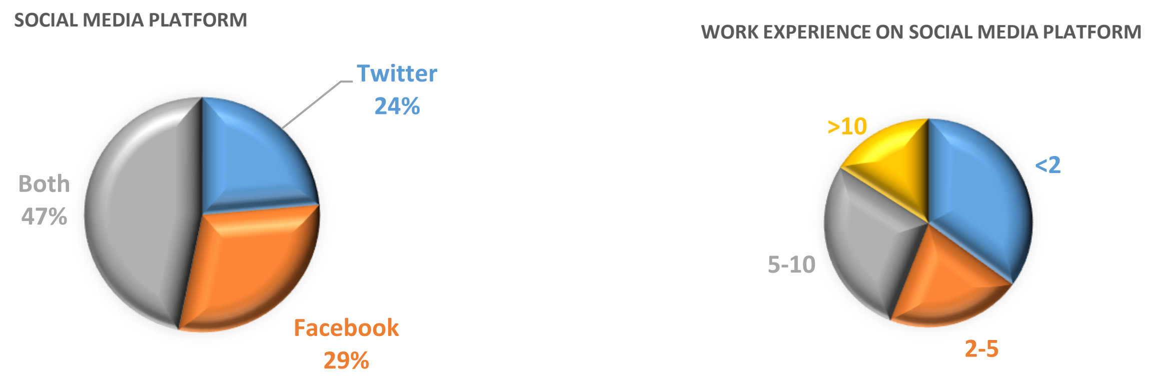

Figure 2: (a) Worked on Social Media Platform and (b) Work Experience Distribution

The 47% of the scientist had worked on both the platform, whereas 29% had their hands on Facebook. Traditional Twitter analytics share 24%. Work experience less than 2 years share 35% whereas between 5-10 years share 28% and 2-5 years share 21%. The least of 16% share is among the work experience more than 10 years. New trends in social media analytics have provide wide opportunities.

Bar Charts was used to find the initial trends of the five factors. To identify the trends the 10-point scale was modifies as a 5-point Likert's scale. Following graph reflected the facts and figures of all the respondents.

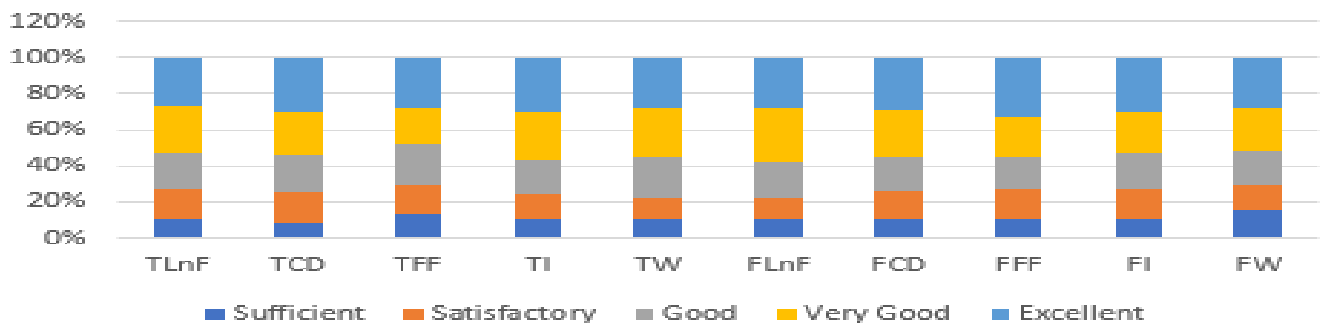

Figure 3: Initial trends values for the given five factors.

Figure 3: Initial trends values for the given five factors.

Above graph indicated that there is neck to neck fight between all factors. In Twitter, Influencers seems to the best one followed by Community detection as well as whistle blower. On the other hand, in Facebook concept of Leader and followers stands the best one. Whistle blowers share least popularity among Facebook and Friend of Friend among Twitter.

Pay off values based on Regression Model

Following two tables provide the payoff matrices in the two cases under consideration.

Table 1: When two players do not share any information

		Player B (Facebook)				
		Leader & Followers	Community Detection	Friend of Friend	Influencers	Whistle Blowers
Player A Twitter	Leader & Followers	(6.06,6.18)	(5.61,5.45)	(5.58,5.55)	(5.92,5.81)	(6.28,5.98)
	Community Detection	(5.68,5.81)	(6.07,5.95)	(5.90,5.89)	(6.26,6.18)	(6.15,5.81)
	Friend of Friend	(4.60,5.39)	(5.42,5.80)	(5.58,6.06)	(5.11,5.53)	(4.99,5.10)
	Influencers	(6.23,6.29)	(5.96,5.80)	(5.78,5.72)	(6.21,6.08)	(6.21,5.83)
	Whistle Blowers	(5.52,5.65)	(5.98,5.87)	(5.50,5.47)	(5.39,5.22)	(5.86,5.47)

Table 2: When two players share any information

		Player B (Facebook)				
		Leader & Followers	Community Detection	Friend of Friend	Influencers	Whistle Blowers
Player A (Twitter)	Leader & Followers	(6.49,6.60)	(6.49,6.43)	(6.49,6.55)	(6.49,6.44)	(6.49,6.25)
	Community Detection	(6.50,6.60)	(6.50,6.43)	(6.50,6.55)	(6.50,6.44)	(6.50,6.25)
	Friend of Friend	(6.13,6.60)	(6.13,6.43)	(6.13,6.55)	(6.13,6.44)	(6.13,6.25)
	Influencers	(6.56,6.60)	6.56,6.43)	(6.56,6.55)	(6.56,6.44)	(6.56,6.25)
	Whistle Blowers	(6.49,6.60)	(6.49,6.43)	6.49,6.55)	(6.49,6.44)	(6.49,6.25)

Optimisation of Strategies

Case 1: When both the players do not share their information

When Twitter is major and Facebook is minor, the value of the game is 5.86.

An optimal strategy for Twitter is: (0, 0.67, 0, 0.33, 0)

An optimal strategy for Facebook is: (0.18, 0, 0.82, 0, 0)

The value of the game is 5.86 i. e. it is on an average seems to be Good one. The Twitter shares the highest probability of success 67% with Leader & Followers and its Friend of Friend, share the probability of success with 33%. The Twitter remaining factors should be improved as compared to the Facebook. Similarly, Facebook, Friend of Friend is much stronger with the highest probability of 82% and also its Leader & Followers, shares the Probability of 18% of its success.

When Facebook is major and Twitter is minor, the value is of the game is 5.72.

An optimal strategy for Facebook is: (0.34, 0.46, 0.20, 0, 0)

An optimal strategy for Twitter is: (0.31, 0, 0.37, 0, 0.32)

The value of the game is 5.72, i.e. it is on an average seems to be Good one. The Facebook community detection shares the probability of success of 46%, Leader & Followers with 34% and Friend of Friend with 20%. On the other hand, Twitter in the minor case shares the highest probability of 37% with Community detection. Friend of friend and Whistle blower shares almost one third of their respective probability of success.

Case 2: When both the players share their information

When Twitter is major and Facebook is minor, the value of the game is 6.56.

An optimal strategy for Twitter is: (0,0,0,1,0)

An optimal strategy for Facebook is: (1,0,0,0,0)

In this case saddle point exists and it's a pure game where Twitter stands best in its Influencer whereas Facebook in its Leader & Followers.

When Facebook is major and Twitter is minor, the value of the game is 6.6.

An optimal strategy for Facebook is: (1, 0, 0, 0, 0)

An optimal strategy for Twitter is :(1, 0, 0, 0, 0)

In this case also saddle point exists and it is a pure game where both Facebook and Twitter stand best in their respective parameter of Leader & Followers.

III.CONCLUSION

This study intent to determine how the top two social network competitors differentiate their common strategies in the Indian market of entertainment. Some of the major findings are summarized as follows-

- Game theory model provides a self-reliant, decision-making tool to optimize the strategies against the business competitor.
- In case of the consumer's responses, data analytical approach appears to be the best approach to find out the trends among the consumers.
- The comparative bar chart reflects the initial trend of the neck-to-neck results in all the five factors under consideration.
- Regression model in this study served as a medium to find out the payoff values.
- In case when the information is not shared among the two players, it is found to be more likely to have a game of mixed strategies. On the other hand, if the two competitors share some of their information, the game is as good as a pure strategic game.
- The values of the Game in case of mixed strategies found to be 5.86 and 5.72. Also, in case of pure strategies, 6.56 and 6.6, indicates and good deal between these two players, where Facebook stands marginally high as compared to Twitter.

This study can be extended as a three-person game where the third player WhatsApp or say Instagram, can optimize its strategies as compared to these two top competitors. Thus, this study can help out the other players of this domain to plan for their strategies.

IV. REFERENCES

[1] . Wu He, Shenghua Zha and Ling Li, "Social media competitive analysis and text mining: A case study in the pizza industry", International Journal of Information Management, Volume 33, Issue 3, Pages 464-472, June 2013.

[2] . Alex Yaw Adom, Israel Kofi Nyarko and Gladys Narki Kumi Som, "Competitor Analysis in Strategic Management: Is it a Worthwhile Managerial Practice in Contemporary Times?", Journal of Resources Development and Management, Vol.24, 2016.

[3] . N. V. Vaidya, and N. N. Kasturiwale, “Solving Game Problems Using a Quick Simplex Algorithm a New Method”, International Journal of Latest Trend Math, Vol-4, No 1,165- 182, 2014.

[4] . Satyajit S. Uparkar, Khushbu R. Asati, Prachi U. Sahare, Nalini V. Vaidya, "Data Analytics of Social Networking sites using Game Theory Model", International Journal of Engineering and Advanced Technology, Volume-8 Issue-6S3, September 2019, 1046-1050.

[5] . Sajedeh Norozpour and Mehdi Safaei, “An Overview on Game Theory and Its Application”, IOP Conf. Series: Materials Science and Engineering 993, 2020.

[6] . Anuja Arora, Aman Srivastava and Shivam Bansal, "Business competitive analysis using promoted post detection on social media " Journal of Retailing and Consumer Services", Volume 54, May 2020.

[7] . T Spiliotopoulos, I Oakley “ An exploration of motives and behavior across Facebook and Twitter” Journal of Systems and Information …, 2020 - emerald.com

[8] . Jayakumar Subramanian Aditya Mahajan Aditya A. Paranjape “On Controllability of Leader-Follower Dynamics over a Directed Graph” http://www.ece.mcgill.ca › conference › 2018-cdc

[9] . Sanket Chobe1and Justin Zhan2 “Advancing community detection using Keyword Attribute Search” https://journalofbigdata.springeropen.com/articles/10.1186/s40537-019-0243-y

[10] . Manishka Gautam, Himanshu Sharma,Mayank Chauhan,Himanshu Shukla, Nidhi Tawra, “Friend Suggestion using Graph” International Journal of Recent Technology and Engineering (IJRTE) ISSN: 2277-3878, Volume-8 Issue-6, March 2020

[11] . Satyajit S. Uparkar, Pranali R. Dandekar and Rajeshri A. Puranik, “Identification of Influencers in Social Networking Site using Graph Database”, International Journal of Novel Research in Engineering, Science & Technology Volume-1, Issue-2, November 2016.

[12] . Shalin Hai-Jew, “Social World Sensing via Social Image Analysis from SocialMedia”, April15,2020, https://kstatelibraries.pressbooks.pub/socialworldsensing/chapter/socimageryblowingwhistles-4/

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Significance of Meruprastar

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ABSTRACT
In this paper we shall discuss a method of finding any power of any number without multiplying by it. The method is known as "Meruprastar". It is also known as "Pascal's triangle". In fact, before hundreds of years of Pascal the method was known and used in India as Meruprastar.
Keywords: Meru (mountain), Prastar (expansion), Meruprastar, Indices rule

I. INTRODUCTION

A number multiplied by itself gives its square. We know the method of "Yavadunam" and the method of "Duplex" to find the square of a number directly without multiplying by itself. When a square of a number is multiplied by the number itself, we get the value of the cube of the number. We know the method of "Yaqvadunam" and the method of ratio to find the cube of the number without multiplying by the number itself.

We shall discuss a method of finding any power of any number without multiplying by it. The method is known as "Meruprastar". It is also known as "Pascal's triangle". In fact, before hundreds of years of Pascal the method was known and used in India as Meruprastar. Its shape is like a mountain and so it is known as Meru (Mountain) prastar (expansion).

It is useful in finding the coefficients of the n-power of (1 + x) means (1 + x)n, (x + 1)n and (x + y)n. It helps to find the values of different powers of integer numbers.

II. DEVELOPMENT OF MERUPRASTAR

First, we place 1 in the middle. Now we consider zeros on its both sides. Now we add the adjacent integers and get 1 & 1, which we place in the second row. Now we consider zeros on both ends and add the adjacent numbers and place the additions in third row as shown below. Continuing the process of placing zeros on both ends and adding adjacent numbers we get the following structure. It is known as Meruprastar.

																		n
								1										**0**
							1		**1**									**1**
						1		**2**		**1**								**2**
					1		**3**		**3**		**1**							**3**
				1		**4**		**6**		**4**		**1**						**4**
			1		**5**		**10**		**10**		**5**		**1**					**5**
		1		**6**		**15**		**20**		**15**		**6**		**1**				**6**
	1		**7**		**21**		**35**		**35**		**21**		**7**		**1**			**7**

Now the question is whether we can get the coefficients of the row related to any positive integer value of n directly or we need to perform all the consequent rows and then we get the coefficients of given value of n. The obvious answer is yes, we can get the coefficients of any row related to any value of n.

Illustration 1: Get the coefficients of the row related to n = 5.

In the first row writes the integers from 5 to 1 in descending order. In the next row below these, write the integers from 1 to 5 in ascending order. Place 1 as first coefficient.

Multiply by the integer in first row in the first column and divide by the integer in the second row below it. This will be second coefficient.

Now multiply this coefficient by the next integer in the first row and divide by the integer below it and get the third coefficient and continue this process till you get the last coefficient 1.

See the calculations in the table below and confirm the method.

×		5	4	3	2	1
÷		1	2	3	4	5
Calculations	1	$\frac{1 \times 5}{1}$	$\frac{5 \times 4}{2}$	$\frac{10 \times 3}{3}$	$\frac{10 \times 2}{4}$	$\frac{5 \times 1}{5}$
Coefficients	1	5	10	10	5	1

Illustration 2: Get the coefficients of the row related to n = 7.

×		7	6	5	4	3	2	1
÷		1	2	3	4	5	6	7
Calculations	1	$1 \times \frac{7}{1}$	$7 \times \frac{6}{2}$	$21 \times \frac{5}{3}$	$35 \times \frac{4}{4}$	$35 \times \frac{3}{5}$	$21 \times \frac{2}{6}$	$7 \times \frac{1}{7}$
Coefficients	1	7	21	35	35	21	7	1

In general, for getting the coefficients related to a natural number n, in the first row we place the natural numbers starting from n up to 1 in descending order and below them in second row we place the natural numbers starting from 1 up to n in ascending order. Then in third row we place one and then in next column we multiply it by the number in first row and divide by the number in the second row. The value on simplification will give the next coefficient.

Now we multiply this value by the number of the first row in the next column and divide by the number below it in the second row and get the successive coefficient. The process is continued till we get the last coefficient 1. Thus, we get all the coefficients of the row related to n.

A. EXPANSION OF $(x + 1)^n$

Write down the coefficients of the row of Meruprastar related to n. Now multiply last coefficient on right by 1 and then go on multiplying the consecutive coefficients by x, x^2, x^3, … and last by x^n. Now adding the products, we get the required expansion as

$(x + 1)^n = x^n + n x^{n-1} + \frac{n(n-1)}{2} x^{n-2} + \dots + \frac{n(n-1)}{2} x^2 + n x + 1.$

Illustration 1: Let us try for expansion of $(x + 1)^5$ applying this method.

The coefficients of the row related to n = 5 in Meruprastar are 1 5 10 10 5 1.

Multiplying them consecutively from right by 1, x, x^2, x^3, x^4 and x^5 we get $1x^5$, $5x^4$, $10x^3$, $10x^2$, $5x$ and 1.

Now adding them we get $(x + 1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$.

Illustration 2: Let us try to get the expansion of $(x + 1)^7$.

The coefficients of row related to n = 7 in Meruprastar are 1 7 21 35 35 21 7 1.

From right multiplying them consecutively by 1, x, x^2, x^3, x^4, x^5, x^6 and x^7, we get

$1x^7$, $7x^6$, $21x^5$, $35x^4$, $35x^3$, $21x^2$, $7x$ and 1.

Now adding them we get

$(x + 1)^7 = x^7 + 7x^6 + 21x^5 + 35x^4 + 35x^3 + 21x^2 + 7x + 1.$

B. EXPANSION OF $(1 + x)^n$

Write down the coefficients of the row of Meruprastar related to n. Now multiply last coefficient on left by 1 and then go on multiplying the consecutive coefficients by x, x^2, x^3, …… and last by x^n. Now adding the products, we get the required expansion as

$(1+x)^n = 1 + n x + \frac{n(n-1)}{2} x^2 + \dots\dots + \frac{n(n-1)}{2} x^{n-2} + n x^{n-1} + x^n$

Illustration 1: Let us try for expansion of $(1 + x)^4$ applying this method.

The coefficients of the row related to n = 4 in Meruprastar are 1 4 6 4 1.

Multiplying them consecutively from left by 1, x, x^2, x^3 and x^4, we get 1, $4x$, $6x^2$, $4x^3$ and x^4.

Now adding them we get

$(1+ x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$

Illustration 2: Let us try for expansion of $(1 + mx)^4$ applying this method.

The coefficients of the row related to n = 4 in Meruprastar are 1 4 6 4 1.

Multiplying them consecutively from left by 1, mx, $(mx)^2$, $(mx)^3$ and $(mx)^4$, we get 1, $4mx$, $6m^2x^2$, $4m^3x^3$ and m^4x^4. Now adding them we get

$(1 + mx)^4 = 1 + 4mx + 6m^2x^2 + 4m^3x^3 + m^4x^4$

Illustration 3: Let us try to get the expansion of $(1 + x)^6$.

The coefficients of row related to n = 6 in Meruprastar are 1 6 15 20 15 6 1.

From left multiplying them consecutively by 1, x, x^2, x^3, x^4, x^5 and x^6. we get 1, $6x$, $15x^2$, $20x^3$, $15x^4$, $6x^5$, and $1x^6$.

Now adding them we get

(1 + x)6 = 1 + 6 x + 15 x2 + 20 x3 + 15 x4 + 6 x5 + x6.

III. APPLICATIONS

A. EXPANSION OF (x + y)n

Write down the coefficients of the row of Meruprastar related to n. Now multiply last coefficient on right by 1 and then go on multiplying the consecutive coefficients by x, x2, x3, …… and last by xn. Now multiply last coefficient on left by 1 and then go on multiplying the consecutive coefficients by y, y2, y3, …… and last by yn. Now adding the products, we get the required expansion as

(x + y)n = xn + n xn-1 y + □((n (n-1))/2) xn – 2 y2 + … + □((n (n-1))/2) x2 yn - 2 + n x yn - 1 + yn

Illustration 1: Let us try for expansion of (x + y)5 applying this method.

The coefficients of the row related to n = 5 in Meruprastar are 1 5 10 10 5 1.

Multiplying them consecutively from right by 1, x, x2, x3, x4 and x5 , we get 1 x5, 5 x4, 10 x3 , 10 x2 , 5 x and 1.

Now multiplying them from left by 1, y, y2, y3 , y4 and y5 , we get x5 , 5 x4 y , 10 x3y2 , 10 x2 y3, 5 x y4 and 1 y5 .

Now adding them we get

(x + y)5 = x5 + 5 x4 y + 10 x3 y2 + 10 x2 y3 + 5 x y4 + y5

Illustration 2: Get expansion of (3x+2y)4 applying this method.

The coefficients of the row related to n = 4 in Meruprastar are 1 4 6 4 1.

Multiplying them consecutively from right by 1, 3x, (3x)2, (3x)3 and (3x)4, and then multiplying them consecutively from left by 1, 2y, (2y)2, (2y)3 and (2y)4 , we get

81x4, 4 (27 x3) (2y), 6 (9x2) (4y2), 4 (3x) (8 y3) and 16 y4.

Now simplifying and adding we get

(3x + 2y) 5 = 81 x4 + 216 x3 y + 216 x2 y2 + 96 x y3 + 16 y4.

Illustration 3: Get expansion of (5x2+7y3)3 applying this method.

The coefficients of the row related to n = 3 in Meruprastar are 1 3 3 1.

Multiplying them consecutively from right by 1, 5x 2, (5x2)2 and (5x2)3 and then multiplying them consecutively from left by 1, 7y3, (7y3)2 and (7y3)3, we get

125 x6, 3 (25 x4) (7 y3), 3 (5x2) (49 y6) and 343y3.

Now simplifying and adding we get

(5x2 + 7y3)3 = 125 x6 + 525 x4 y3 + 735 x2 y6 + 343 y9.

B. POWERS OF NUMBERS

Illustration 1: To find the value of 125.

We know that (x + y)5 = x5 + 5 x4 y + 10 x3 y2 + 10 x2 y3+ 5 x y4 + y5

Taking x = 10 and y = 2, we have (x + y)5 = (10 + 2)5 = (12)5 so power of x =10 will turn into the place value, so we need to multiply the coefficients by power of y = 2 only.

Coefficients For n = 5	1	5	10	10	5	1
$\times\ 2^n$	1	2	4	8	16	32
Products	1	10	40	80	80	32

So (12)5 = 1| 10 | 40 | 80 | 80| 32
= 1| 10 | 40 | 80 | 83| (2)
= 1| 10 | 40 | 88 | (32)
= 1| 10 | 48 | (832)
= 1| 14 | (8832) = 1 + 1 | (48832) = 248832

In short this can be easily calculated as –

(12)5 = 1×1 5×2 10×4 10×8 5×16 1×32
= 1 10 40 80 80 32 = 248832.

Illustration 2: To find the value of 215.

Now know that the coefficients are to be multiplied by the powers of 2 from right side, so we can directly calculate in short as

(21)5 = 1×32 5×16 10×8 10×4 5×2 1×1
= 32 80 80 40 10 1 = 4084101.

Note: We can note that if 2 is on unit place means on right, then we need to multiply the coefficients by the powers of 2 from left and if 2 is on tens place means on left, then we need to multiply the coefficients by the powers of 2 from right. Using this we can easily find the value in the next illustration.

Illustration 3: To find the value of 325.

Here 3 is on tens place means on left place so we need to multiply the coefficients by the powers of 3 from right and 2 is on unit place means on right place, so we will have to multiply the coefficients by the powers of 2 from left.

So 325
= 1×1×243 5×2×81 10×4×27 10×8×9 5×16×3 1×32×1
= 243 810 1080 720 240 32 = 243 810 1080 720 (240+3)2
= 243 810 1080 (720+24) 32 = 243 810 (1080+74) 432
= 243 (810+115) 4432 = (243+92) 54432 = 33554432

Illustration 4: To find the value of 534.

Coefficients related to n = 4 in Meruprastar are 1 4 6 4 1 and 5 is on tens place means on left and 3 is on unit place means on left.

So 53 4 = 1×1 ×625 4×3×125 6×9×25 4×27×5 1×81×1
= 625 1500 1350 540 81 = 7890481

Illustration 5: To find the value of 743.

Coefficients related to n = 3 in Meruprastar are 1 3 3 1 and 7 is on tens place means on left and 4 is on unit place means on left.

So $74^3 = 1\times1\times343 \quad 3\times4\times49 \quad 3\times16\times7 \quad 1\times64\times1$

$= 343 \quad 588 \quad 336 \quad 64 = 405224.$

IV. CONCLUSION

In conventional method of finding various powers of a number, the only method of multiplication is used which is difficult for finding the value of higher powers of a number.

The method of Meruprastar is useful in finding the value of various powers of a number without actual multiplication. Similarly, this method is useful for manual calculation.

V. ACKNOWLEDGENT

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VI.REFERENCES

[1] . Jagadguru Swami Bharati Krisna Tirthaji Maharaja: Vedic Mathematics, Motilal Banarasidass, Delhi

[2] . V.G. Unkalkar: Magical World of Mathematics, Vandana Publishers, Banglore

[3] . Shriram Chauthaiwale and Dr. Ramesh Kollru: Enjoy Vedic Mathematics, Sri Sri Publication, Banglore

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Analytical Solutions of the Fokker-Planck Equation by Laplace Decomposition Method

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ABSTRACT

In this paper, the Laplace decomposition method is employed to obtain an analytical solution of the Fokker-Planck equation. Solutions are obtained in series form. Exact solutions are achieved by the known form of the series. The result shows the efficiency, applicability, and accuracy of the Laplace decomposition method in solving linear and nonlinear differential equations.

Keywords: Laplace decomposition method, Fokker-Planck equation, Kolmogorov equation, Adomian polynomials, partial differential equations.

I. INTRODUCTION

Partial differential equations are played an elementary role in describing many important phenomena in the field of science and engineering such as plasma physics, biology, anharmonic lattices, chemistry, cosmology, optics fiber, finance, quantum mechanics, and so on. Solution of the partial differential equations are play important role in understanding different physical phenomena. Many methods are available in the literature to study PDEs reference therein [1-5].

The Fokker-Planck equation was used by Fokker and Planck to describe the Brownian motion of particles [6]. K. Morgan and D. Paganin used FPE to describe the drift and diffusion of the periodic or structured illumination used in phase-contrast x-ray imaging with grating, to better understand any cross-talk between attenuation, phase, and dark-field x-ray signals [7]. Many important phenomena in various fields are modeled by FPE such as population dynamics, circuit theory, electrodynamics, biophysics, neurosciences, polymer physics, psychology, surface physics, chemical physics, quantum optics, laser physics, and marketing [1]. M. Jawary applies NIM to find the analytical solution of FPE [8]. In [9] author solved FPE with the help of ADM. Z. Korpinar and co-authors implemented the Laplace homotopy analysis method for the fractional model of the Fokker-Planck equation [10]. H. Aminikhan and A. Jamalian combine Laplace transform and New Homotopy Perturbation Method to obtain the solution of nonlinear Fokker-Planck equation [11]. In [12] author obtained numerical solutions of fractional Fokker-Planck equation with the help of the Laplace transform method. A. Hemeda and E. Eladdad employed New Iterated Method for the study of FPE [13].

In the present article, we employed the Laplace Decomposition Method for FPE which was introduced by Khuri in 2001[14]. Further, Hussain and Khan modified this technique in 2010 [15]. A. Kumar and R. Pankaj obtained solitary wave solutions of coupled nonlinear partial differential equations by employing LDM [16]. S. Handibag and B. Karande used LDM to solve the fifth-order KdV equation [17]. S. Handibag and R. Wayal implemented LDM to obtain the solutions of the Drinfield Sokolov (DS) system, the coupled Burger's equation, and the Cauchy problem [18].

II. FOKKER-PLANCK EQUATION

The general form of the Fokker-Planck equation with a variable x is [1,6]

$$\frac{\partial u}{\partial t} = \left(-\frac{\partial}{\partial x}A(x) + \frac{\partial^2}{\partial x^2}B(x)\right)u, \quad (2.1)$$

subject to the initial condition

$$u(x,0) = f(x), \qquad x \in \mathbb{R}. \qquad (2.2)$$

Where $u(x,t)$ is unknown, $A(x)$ is called the drift coefficient and $B(x) > 0$ is diffusion coefficient. The drift and diffusion coefficients may depend on time also, i.e.

$$\frac{\partial u}{\partial t} = \left(-\frac{\partial}{\partial x}A(x,t) + \frac{\partial^2}{\partial x^2}B(x,t)\right)u. \qquad (2.3)$$

Eq. (2.1) is a linear second-order partial differential equation of parabolic type which represents the equation of motion for the distribution function $u(x,t)$. It is also called the forward Kolmogorov equation. A similar partial differential equation is a backward Kolmogorov equation that is in the form [1,6]

$$\frac{\partial u}{\partial t} = -\left(A(x,t)\frac{\partial}{\partial x} + B(x,t)\frac{\partial^2}{\partial x^2}\right)u. \qquad (2.4)$$

The general form of the nonlinear Fokker-Planck equation for one variable is written as:

$$\frac{\partial u}{\partial t} = \left(-\frac{\partial}{\partial x}A(x,t,u) + \frac{\partial^2}{\partial x^2}B(x,t,u)\right)u. \ (2.5)$$

III. LAPLACE DECOMPOSITION METHOD

Consider the general form of the NLPDE

$$\mathcal{L}\mathrm{u}(\mathrm{x},\mathrm{t}) + \mathcal{R}\mathrm{u}(\mathrm{x},\mathrm{t}) + \mathcal{N}\mathrm{u}(\mathrm{x},\mathrm{t}) = 0, \quad (3.1)$$

with initial condition

$$u(\mathrm{x},0) = f(\mathrm{x}).$$

where $\mathcal{L} = \frac{\partial}{\partial \mathrm{t}}$, $\mathcal{R}$ is general linear operator, $\mathcal{N}u$ is the nonlinear term. Taking Laplace transform of (3.1) w. r. t. t

$$su(\mathrm{x},\mathrm{s}) - \mathrm{u}(\mathrm{x},0) = -\mathfrak{L}_{\mathrm{t}}[\mathcal{R}\mathrm{u}(\mathrm{x},\mathrm{t}) + \mathcal{N}\mathrm{u}(\mathrm{x},\mathrm{t})],$$

$$u(\mathrm{x},\mathrm{s}) = \frac{f(\mathrm{x})}{\mathrm{s}} - \frac{1}{\mathrm{s}}\mathfrak{L}_{\mathrm{t}}[\mathcal{R}\mathrm{u}(\mathrm{x},\mathrm{t}) + \mathcal{N}\mathrm{u}(\mathrm{x},\mathrm{t})].$$

Taking inverse Laplace transform

$$\mathrm{u}(\mathrm{x},\mathrm{t}) = f(\mathrm{x}) - \mathfrak{L}_{\mathrm{t}}^{-1}\left\{\frac{1}{\mathrm{s}}\mathfrak{L}_{\mathrm{t}}[\mathcal{R}\mathrm{u}(\mathrm{x},\mathrm{t}) + \mathcal{N}\mathrm{u}(\mathrm{x},\mathrm{t})]\right\} \qquad (3.2)$$

Represent solution of the equation (3.1) in an infinite series form,

$$\mathrm{u}(\mathrm{x},\mathrm{t}) = \sum_{\mathrm{n}=0}^{\infty}\mathrm{u}_{\mathrm{n}}(\mathrm{x},\mathrm{t}). \qquad (3.3)$$

The nonlinear term is represented by an infinite series of the so-called Adomian polynomials

$N\mathrm{u} = \sum_{\mathrm{n}=0}^{\infty} \mathcal{A}_{\mathrm{n}}$. (3.4)

The Adomian polynomials are generated with the help of the following relation $\mathcal{A}_{\mathrm{n}} = \frac{1}{\mathrm{n}!}\frac{\mathrm{d}^{\mathrm{n}}}{\mathrm{d}\lambda^{\mathrm{n}}}\left[\mathrm{N}\left(\sum_{\mathrm{k}=0}^{\infty} \lambda^{\mathrm{k}} \mathrm{u}_{\mathrm{k}}\right)\right]_{\lambda=0}$, $\mathrm{n} \geq 0$. (3.5)

From equation (3.2) to (3.4) $\sum_{\mathrm{n}=0}^{\infty} u_{\mathrm{n}}(\mathrm{x},\mathrm{t}) = \mathrm{f}(\mathrm{x}) - \mathfrak{L}_{\mathrm{t}}^{-1}\left\{\frac{1}{\mathrm{s}}\mathfrak{L}_{\mathrm{t}}[\mathcal{R}\sum_{\mathrm{n}=0}^{\infty} \mathrm{u}_{\mathrm{n}}(\mathrm{x},\mathrm{t}) + \sum_{\mathrm{n}=0}^{\infty} \mathcal{A}_{\mathrm{n}}]\right\}$. (3.6)

By applying the LDM for equation (3.6) the following recurrence relation for the determination of the components $u_{n+1}(x,t)$ are obtained:

$$u_0(\mathrm{x},\mathrm{t}) = f(\mathrm{x})$$

$$u_{\mathrm{n}+1}(\mathrm{x},\mathrm{t}) = -\mathfrak{L}_{\mathrm{t}}^{-1}\left\{\frac{1}{\mathrm{s}}\mathfrak{L}_{\mathrm{t}}[\mathcal{R}(\mathrm{u}_{\mathrm{n}}) + \mathcal{A}_{\mathrm{n}}]\right\},$$

$$\mathrm{n} = 0,1,2,3,\dots . \qquad (3.7)$$

Based on the LDM solution of equation (3.1) is

$$u(x,t) = \lim_{n\to\infty} \phi_n(x,t), \qquad (3.8)$$

where $\phi_n(x,t) = \sum_{\mathrm{k}=0}^{\mathrm{n}} \mathrm{u}_{\mathrm{k}}(\mathrm{x},\mathrm{t})$.

IV. ILLUSTRATION

Example 1: Consider nonlinear Fokker-Planck equation (2.5) with [1,8,11]

$$A(x,t,u) = 4\frac{u}{x} - \frac{x}{3}, \qquad (4.1)$$

$$B(x,t,u) = u,$$

and the initial condition

$$u(x,0) = x^2 = f(x).$$

$$\frac{\partial u}{\partial t} = \left(-\frac{\partial}{\partial x}\left(4\frac{u}{x} - \frac{x}{3}\right) + \frac{\partial^2}{\partial x^2}u\right)u,$$

$$\frac{\partial u}{\partial t} = \frac{1}{3}u + \frac{x}{3}u_x + \frac{4}{x^2}u^2 - \frac{8}{x}uu_x + 2u_x^2 + 2uu_{xx}.$$

Applying LDM algorithm, we get the following recursive relation:

$$u_0(x,t) = x^2,$$

$$u_{n+1}(x,t) = \mathfrak{L}_{\mathrm{t}}^{-1}\left\{\frac{1}{s}\mathfrak{L}_{\mathrm{t}}\left[\frac{1}{3}u_n + \frac{x}{3}u_{n_x} + \frac{4}{x^2}\mathcal{A}_n - \frac{8}{x}\mathcal{B}_n + 2\mathcal{C}_n + 2\mathcal{D}_n\right]\right\}, n = 0,1,2,\dots \qquad (4.2)$$

Where $\mathcal{A}_n, \mathcal{B}_n, \mathcal{C}_n, and\ \mathcal{D}_n$ are Adomian polynomials are easily calculated by using Eq. (3.5). Few Adomian polynomials are:

$\mathcal{A}_0 = u_0^2$, $\mathcal{B}_0 = u_0 u_{0_x}$,

$\mathcal{C}_0 = {u_0}_x^2$ $\mathcal{D}_0 = u_0 u_{0_{xx}}$,

$\mathcal{A}_1 = 2u_0u_1$, $\mathcal{B}_1 = u_0u_{1_x} + u_1u_{0_x}$,

$\mathcal{C}_1 = 2u_{0_x}u_{1_x}$, $\mathcal{D}_1 = u_0u_{1_{xx}} + u_1u_{0_{xx}}$

$\mathcal{A}_2 = u_1^2 + 2u_0u_2$, $\mathcal{B}_2 = u_0u_{2_x} + u_1u_{1_x} + u_2u_{0_x}$,

$\mathcal{C}_2 = u_{1_x}^2 + 2u_{0_x}u_{2_x}$,

$$\mathcal{D}_2 = u_0u_{2_{xx}} + u_1u_{1_{xx}} + u_2u_{0_{xx}},$$

and so on. Using the recurrence relation defined in Eq. (4.2), we obtain

$$u_1(x,t) = x^2t,$$

$$u_2(x,t) = x^2 \frac{t^2}{2!},$$
$$u_3(x,t) = x^2 \frac{t^3}{3!}.$$

In a similar manner, we can calculate the remaining components of the series. Therefore, according to Eq. (3.3) we have

$u(x,t) = x^2 \left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots\right).$

This has the closed form

$$u(x,t) = x^2 e^t.$$

Which is the exact solution of Eq. (4.1). Result can be verified through substitution.

Example 2: Consider Eq. (2.1) with initial condition [1,8]

$$u(x,0) = x, \qquad x \in \mathbb{R}$$

Let $A(x) = -1, B(x) = 1$ then linear FPE is

$$\frac{\partial u}{\partial t} = u_x + u_{xx}. \qquad (4.3)$$

By the LDM, according to Eq. (3.7) recurrence relation for example 2 is:

$$u_0(x,t) = x,$$

$u_{n+1} = \mathfrak{L}_t^{-1}\left\{\frac{1}{s}\mathfrak{L}_t\left[u_{n_x} + u_{n_{xx}}\right]\right\}, \qquad n = 0,1,2, \dots .$

Therefore, the first few components of the series solution are:

$$u_1(x,t) = t,$$
$$u_i(x,t) = 0, i \geq 2.$$

Therefore, from Eq. 3.3 solution of the considered problem is

$u(x,t) = x + t,$

which is the exact solution of Eq. (4.3).

Example 3: Consider Eq. (2.3) with the initial condition [1,8]

$$u(x,0) = \sinh(x), x \in \mathbb{R}. \qquad (4.4)$$

In this case, we consider A and B depends on x and t.

$$A(x,t) = e^t \coth(x)\cosh(x) + e^t \sinh(x) - \coth(x),$$
$$B(x,t) = e^t \cosh(x).$$

Proceeding as before, components of the series by LDM are:

$$u_0(x,t) = \sinh(x),$$
$$u_1(x,t) = \sinh(x)\, t,$$
$$u_2(x,t) = \sinh(x)\, \frac{t^2}{2!},$$
$$u_3(x,t) = \sinh(x)\, \frac{t^3}{3!},$$

and so on.

Consequently, the series solution is

$$u(x,t) = \sinh(x)\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots\right).$$

According to Eq. (3.8) exact solution of equation (2.3) with initial condition (4.4) is

$$u(x,t) = \sinh(x)\, e^t.$$

Example 4: Consider linear FPE with variable coefficient in Eq. (2.1) [12]

$$A(x) = x, B(x) = \frac{x^2}{2}.$$

With initial condition $f(x) = x,\ x \in \mathbb{R}.$ (4.5)

LDM yield the following recurrence relation:

$$u_0(x,t) = x,$$
$$u_{n+1}(x,t) = \mathfrak{L}_t^{-1}\left\{\frac{1}{s}\mathfrak{L}_t\left[x u_{n_x} + \frac{x^2}{2}\, u_{n_{xx}}\right]\right\}, \qquad n \geq 0.$$

Thus, components of the series are:

$$u_1(x,t) = xt,$$
$$u_2(x,t) = x\frac{t^2}{2!},$$
$$u_3(x,t) = x\frac{t^3}{3!},$$

By the same procedure rest of the components are easily calculated and

$$\phi_n(x,t) = x\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots + \frac{t^n}{n!}\right).$$

Closed-form of the solution is $u(x,t) = xe^t$, which is the exact solution of Eq. (3.1) with initial condition (4.5).

Example 5: Now consider backward Kolmogorov equation [2.4] with the initial condition [1,8]

$$u(x,0) = x + 1, \qquad (4.6)$$

and $A(x,t) = -(x+1),$ $B(x,t) = x^2 e^t.$

Proceeding as before, the first components of the series are

$$u_0(x,t) = x + 1,$$
$$u_1(x,t) = (x+1)\, t,$$
$$u_2(x,t) = (x+1)\frac{t^2}{2!},$$

$u_3(x,t) = (x+1)\ \frac{t^3}{3!},$

and so on. Therefore, LDM solution is

$$u(x,t) = (x+1)\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots\right),$$

Closed-form of the above series gives an exact solution of the backward Kolmogorov equation:

$u(x,t) = (x+1)e^t.$

V. CONCLUSION

The Laplace decomposition method has been successfully implemented for the Fokker-Planck equation. Approximate solutions obtained are in an infinite series form which are rapidly converges to the exact solutions. Method decreases the volume of the calculations also it does not require linearization, perturbation, or discretization. By applying the LDM, one can construct the approximate as well as exact solutions to the linear and nonlinear differential and integral equations.

VI.REFERENCES

[1] . Prof. M. Tataria, M. Dehghana, M. Razzaghib, Application of the Adomian decomposition method for the Fokker-Planck equation, Math. And comp. modelling, Volume 45 (2007), 639-650.

[2] . J. Huan He, H. Hong Wu, Variational iteration method: new development and applications, Comp. and Math. with appl., Volume 54, Issues 7-8 (2007) 881-894.

[3] . F. Ayaz, Solutions of the system of differential equations by differential transform method, Appl. math. and comp., Volume 147 (2004) 547-567.

[4] . J. Biazar, M. Eslami, A new homotopy perturbation method for solving system of partial differential equations, Comp. and math with appl., Volume 62, Issue 1 (2011) 225-234.

[5] . M. Elsayed, A. Khaled, The homogeneous balance method and its applications for finding the exact solutions for nonlinear evolution equations, Italian J. of pure and appl. Math. Volume 33 (2014) 307-318.

[6] . H. Risken, The Fokker-Plank equation, Method of solution and applications, Springer Verlag, Beelin, Heidelberg, 1989.

[7] . K. Morgan, D. Paganin, Applying the Fokker-Planck equation to grating-based x-ray phase and dark-field imaging, Scientific reports, 9, Article no. 17465(2019).

[8] . M. Jawary, An efficient iterative method for solving the Fokker-Planck equation, Results in Physics 418 (2016).

[9] . M. Tataria, M. Dehghana, M. Razzaghib, Application of the Adomian decomposition method for the Fokker-Planck equation, Math. and comp. modelling, Volume 45 (2007), 639-650.

[10] . Z. Korpinar, M. Inc, D. Baleanu, On the fractional model of Fokker-Planck equations with two different operators, AIMS Mathematics, Volume 5, Issue 1(2019), 236-248.

[11] . H. Aminikhan and A. Jamalian, A new efficient method for solving the nonlinear Fokker-Planck equation, Scientia Iranica B, Volume 19, Issue 4 (2012), 1133-1139.

[12] . L. Yan, Numerical solutions of Fractional Fokker-Planck equations using Iterative Laplace Transform method, Hindawi Publ. corp. Abstract and Appl. Analysis, Volume 2019, Article ID 465160, 7 pages.

[13] . A. Hemeda and E. Eladdad, New iterated Methods for solving Fokker-Planck Equation, Hindawi Math. Probl. In Eng. Volime 2018, Article ID 6462174, 9 pages.

[14] . S. Khuri, A Laplace decomposition algorithm applied to a class of nonlinear differential equations, Hindawi publishing corp. J. of appl. Math., volume 1, issue 4 (2001), 141-155.

[15] . M. Hussain, M. Khan, Modified Laplace decomposition method, Appl. Math. Sci. volume 4, issue 36(2010), 1769-1783.

[16] . A. Kumar, R. Pankaj, Laplace Decomposition Method to Study Solitary Wave Solutions of Coupled Nonlinear Partial Differential Equation, Int. Scholarly Research Notices, Volume 2012, Article ID 423469.

[17] . S. S. Handibag and B. D. Karande, Existence the Solutions of Some Fifth-Order Kdv Equation by Laplace Decomposition Method, American Journal of Computational Mathematics, Volume 3 (2013) 80-85.

[18] . S. Handibag. R. Wayal, Study of some system of nonlinear partial differential equations by LDM and MLDM, I. J. Sci and research publications, Volume 11, Issue 6 (2021) 449-456.

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Five Dimensional Bianchi Type I Cosmology in *f*(R, T) Gravity

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ABSTRACT

Here, we have investigated the exact solutions of higher five dimensional Bianchi type- I space time in the context of f(R,T) theory of Gravity. In f(R,T)theory of gravity, we have obtained two exact solutions using the assumption of constant deceleration parameter and variation law of Hubble parameter . The first solution gives a singular model for with power law expansion and second gives a non–singular model for with expansion of the universe.

Keywords : f(R,T)theory of gravitation, Bianchi type I, wet dark fluid, deceleration parameter.

I. INTRODUCTION

Accelerating expansion of universe is the most popular issue in the modern day cosmology. It is now proved from observational and theoretical facts that our universe is in the phase of acceleration expansion. The Phenomenon of dark energy and dark matter is also another topic of discussion. Einstein first gave the concept of dark energy and has introduced the positive cosmological constant. But after some time he concluded that it was the biggest mistake in his life. But now a days it is observed that the cosmological constant is suitable for dark energy. In order to explain the current expansion of universe, number of cosmological models has been proposed by some authors. f(T) theory of gravity is one of the such examples which has been developed recently to justify the current expansion of universe. It is interestingly to note that this theory explains the current acceleration without involving dark energy. Ratbay M.[1], M.Sharifet.at. [2], Wei H. et.al.[3], Bamba K. et.at.[4] have studied f(T) theory of gravity. Another interesting modified theory is f(R) theory of gravity in which a general function of Ricci Scalar involves in standard Einstein – Hilbert Lagrangian. Some authors have studied f(R) theory of gravity in different contexts,[5,6,7-20]. Recently Harko et.al.[21] proposed a new generalized theory known as f(R,T) gravity. In f(R,T) gravity, gravitational Lagrangian involves an arbitrary function of the scalar curvature R and the trace of the energy momentum tensor T. Myzokulov [22], Sharif Zubair [23,24], Adhov K.S. [25], L.S. Ladke et al.[26] have studied f(R,T) gravity. Haundjo [27] reconstructed f(R,T) theory of gravity by taking $f(R,T) = f_1(R) + f_2(T)$ and it was observed that, f(R,T) gravity allowed transition of matter from dominated phase to an accelerated phase. Adhav [28], Reddy et.al.[29], Ahemed and

Pradhan [30],Naidu et.al.[31] have observed f(R,T) theory of gravity. Deo and Borsare [32] have studied Bianchi Type I Wet Dark Energy in f(R,T) Gravity.

Higher dimensional cosmological models play an important role in many aspects of early stage of cosmological problems. The study of five dimensional models give an idea that our universe is much smaller at the early stage of evaluation as compared today. Kaluza and Klein [33,34] have done remarkable work in higher dimensional space time. Wesson [35,36], D.R.K. Reddy [37] have studied several aspects of five dimensional space-time. Lorentz and Petzod [38], Ibanez and Verdaguer [39], Adhav et.al. [40] have studied multidimensional cosmological models in general theory of relativity and in other alternative theories of gravitation.

II. FIELD EQUATIONS OF F(R,T) GRAVITY WITH WET DARK ENERGY

In five dimensional space time the action for *f(R,T)* is given by

$$S=\int\left(\frac{1}{16\pi G}f(R,T)+L_m\right)\sqrt{-g}\,d^5x, \tag{2.1}$$

Where *f(R,T)* is an arbitrary function of Ricci scalar R and T trace energy momentum tensor of matter T_{ij}, L_m is matter Lagrangian density for wet dark energy.

The five dimensional field equation in *f(R,T)* theory of gravity are given by

$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}-(\nabla_i\nabla_j-g_{ij}\Box)f(R,T)=kT_{ij}-f_T(R,T)(T_{ij}+\theta_{ij})\ (i,j=1,2,\dots,5), \tag{2.2}$$

Where

$$f_R(R,T)\equiv\frac{\partial f_R(R,T)}{\partial R},\ f_T(R,T)\equiv\frac{\partial f_T(R,T)}{\partial T},\ T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\partial\left(\sqrt{-g}L_m\right)}{\partial g^{ij}},\ \theta_{ij}=-p_{wdf}g_{ij}-2T_{ij} \tag{2.3}$$

$\Box\equiv\nabla^i\nabla_i$, ∇_i is the covariant derivative.

The energy momentum tensor for wet dark fluid yields

$$T_i^j=\left(\rho_{wdf}+p_{wdf}\right)u_iu^j-p_{wdf}\delta_i^j, \tag{2.4}$$

Where u^i is a flow vector satisfying $g_{ij}u^iu^j=1$,

$T_5^5=\rho_{wdf}$, $T_1^1=T_2^2=T_3^3=T_4^4=-p_{wdf}$

Equation of state for wet dark fluid is

$$p_{wdf}=\gamma\left(\rho_{wdf}-\rho^*\right) \tag{2.5}$$

And it is good approximation for many fluids including water, where the internal attraction of the molecules make negative pressure possibly. The parameter γ and ρ^* are taken to be positive with $0\le\gamma\le1$.

Where ρ_{wdf} and p_{wdf} are energy density and pressure of wet dark fluid respectively.

Contracting the field equation (2.2) we get

$$f_R(R,T)R+4\Box f_R(R,T)-\frac{5}{2}f(R,T)=kT-f_T(R,T)(T+\theta) \tag{2.6}$$

From and also the equation (2.2) with (2.3) we have

$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}-\left(\nabla_i\nabla_j-g_{ij}\Box\right)f_R(R,T)=kT_{ij}+f_T(R,T)\left(T_{ij}+p_{wdf}g_{ij}\right) \quad (2.7)$$

Harko et.at. [41] gives three class of models which are

$$f(R,T)=\begin{cases} R+2f(T), \\ f_1(R)+f_2(T), \\ f_1(R)+f_2(R)f_3(T). \end{cases}$$

Out of which we have used $f(R,T)=R+2f(T)$

The equation (2.7) can be written as

$$R_{ij}-\frac{1}{2}Rg_{ij}=kT_{ij}+2f'(T)T_{ij}+\left[f(T)+2p_{wdf}f'(T)\right]g_{ij} \quad (2.8)$$

Overhead prime denotes derivative with respect to T.

For the sake of simplicity we can choose

$f(T)=\lambda T$, Where λ is a constant? (2.9)

III. EXACT SOLUTION OF BIANCHI TYPE – I UNIVERSE IN

Here, we can find exact solutions of five dimension Bianchi type -I space time in *f*(R,T) theory of gravity. The line element of Bianchi type - I model in V_5 is given by

$$ds^2=dt^2-A^2dx^2-B^2dy^2-C^2\left(dz^2+du^2\right), \quad (3.1)$$

Where A, B and C are function of t only.

and the corresponding Ricci Scalar is

$$R=-2\left[\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+2\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+2\frac{\dot{B}\dot{C}}{BC}+2\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{C}^2}{C^2}\right], \quad (3.2)$$

Where overhead dot means derivative with respect to t.

From equations (2.4) - (3.1), we get

$$\frac{\dot{A}\dot{B}}{AB}+2\frac{\dot{B}\dot{C}}{BC}+2\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{C}^2}{C^2}=(k+3\lambda)\rho_{wdf}-2\lambda p_{wdf} \quad (3.3)$$

$$\frac{\ddot{B}}{B}+2\frac{\ddot{C}}{C}+2\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}^2}{C^2}=\lambda\rho_{wdf}-(k+4\lambda)p_{wdf} \quad (3.4)$$

$$\frac{\ddot{A}}{A}+2\frac{\ddot{C}}{C}+2\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{C}^2}{C^2}=\lambda\rho_{wdf}-(k+4\lambda)p_{wdf} \quad (3.5)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}=\lambda\rho_{wdf}-(k+4\lambda)p_{wdf} \quad (3.6)$$

There are four non – linear differential equations consist of five unknown functions namely A, B, C, P_{wdf} and ρ_{wdf}. Hence to find the solutions one more condition is required, so we consider the well known relation between Hubble Parameter H and average scale factor 'a' given as

$H = la^{-n}$, Where $l > 0$ and $n \geq 0$ (3.7)

from equations (3.4) - (3.6), we have

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{2\ddot{C}}{C}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0, \tag{3.8}$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right)\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0, \tag{3.9}$$

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right) = 0, \tag{3.10}$$

On solving above equations, we get

$$\frac{B}{A} = m_1 \exp\left[n_1 \int \frac{dt}{a^4}\right], \tag{3.11}$$

$$\frac{C}{B} = m_2 \exp\left[n_2 \int \frac{dt}{a^4}\right], \tag{3.12}$$

$$\frac{A}{C} = m_3 \exp\left[n_3 \int \frac{dt}{a^4}\right], \tag{3.13}$$

Where n_1, n_2, n_3 and m_1, m_2, m_3 are constants of integration which satisfy the relation

$$n_1 + n_2 + n_3 = 0, \; m_1 m_2 m_3 = 1. \tag{3.14}$$

Using equation (3.11) to (3.13) we get

$$A = ad_1 \exp\left[c_1 \int \frac{dt}{a^4}\right], \tag{3.15}$$

$$B = ad_2 \exp\left[c_2 \int \frac{dt}{a^4}\right], \tag{3.16}$$

$$C = ad_3 \exp\left[c_3 \int \frac{dt}{a^4}\right], \tag{3.17}$$

Where $d_1 = \left(m_1^{-3} m_2^{-2}\right)^{\frac{1}{4}}$, $d_2 = \left(m_1 m_2^{-2}\right)^{\frac{1}{4}}$, $d_3 = \left(m_1 m_2^{2}\right)^{\frac{1}{4}}$, (3.18)

And $c_1 = -\frac{3n_1 + 2n_2}{4}$, $c_2 = \frac{n_1 - 2n_2}{4}$, $c_3 = \frac{n_1 + 2n_2}{4}$, (3.19)

Satisfying the relation

$$d_1 d_2 d_3^2 = 1, \; c_1 + c_2 + 2c_3 = 0 \tag{3.20}$$

IV. IMPORTANT PHYSICAL PARAMETERS

In this section we define some important physical parameters.

The average scale factor a and the volume scale factor V are defined as

$$a=\left(ABC^2\right)^{\frac{1}{4}},\ V=a^4=ABC^2 \tag{4.1}$$

The generalized mean Hubble parameter H is defined by

$$H=\left(\ln a\right)_t=\frac{\dot{a}}{a}=\frac{1}{4}\left[H_x+H_y+H_z+H_u\right], \tag{4.2}$$

Where $H_x=\frac{\dot{A}}{A},\ H_y=\frac{\dot{B}}{B},\ H_z=H_u=\frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x, y, z and u axes respectively.

The mean anisotropy parameter A is given by

$$A=\frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right)^2, \tag{4.3}$$

Where $\Delta H_i=H_i-H$

The expansion scalar θ and shear scalar σ^2 are defined as

$$\theta=u^i_{;i}=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+2\frac{\dot{C}}{C} \tag{4.4}$$

$$\sigma^2=\frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{4.5}$$

Where $\sigma_{ij}=\frac{1}{2}\left[\nabla_j u_i+\nabla_i u_j\right]-\frac{1}{4}\theta g_{ij}$, (4.6)

The deceleration parameter q is the measure of the cosmic accelerated expansion of universe. It is defined as

$$q=-\frac{\ddot{a}a}{\dot{a}^2}. \tag{4.7}$$

The behavior of universe is determined by the sign of q. The positive value of deceleration parameter shows decelerating model while the negative value indicates inflation.

From equation (3.7) and (4.2), we have

$$\dot{a}=la^{1-n}, \tag{4.8}$$

and the equation (4.8) and (4.7) we get $q=n-1$ = constant.

After integrating equation (4.8), we obtain

$$a=\left(nlt+k_1\right)^{1/n},n\neq 0, \tag{4.9}$$

and

$$a=k_2\exp\left(lt\right),n=0, \tag{4.10}$$

Where k_1 and k_2 are constant of integration.

Thus we have two values of the average scale factors which correspond to two different models of the universe.

V. FIVE DIMENSIONAL SINGULAR MODEL WHEN

In this section we study the five dimensional model of the universe for $n\neq 0$

For this singular model average scale factor a given as $a=(nlt+k_1)^{\frac{1}{n}}$

The metric coefficient $A\ B\ C$ are determined as

$$A=d_1(nlt+k_1)^{\frac{1}{n}}\exp\left[\frac{c_1(nlt+k_1)^{\frac{n-4}{n}}}{l(n-4)}\right],\quad n\neq 4 \tag{5.1}$$

$$B=d_2(nlt+k_1)^{\frac{1}{n}}\exp\left[\frac{c_2(nlt+k_1)^{\frac{n-4}{n}}}{l(n-4)}\right],\quad n\neq 4 \tag{5.2}$$

$$C=d_3(nlt+k_1)^{\frac{1}{4}}\exp\left[\frac{c_3(nlt+k_1)^{\frac{n-4}{n}}}{l(n-4)}\right],\quad n\neq 4 \tag{5.3}$$

The mean generalized Hubble parameter and the volume scale factor become

$$H=\frac{l}{nlt+k_1},\ V=(nlt+k_1)^{\frac{4}{n}}. \tag{5.4}$$

The mean anisotropy parameter A turns out to be

$$A=\frac{c_1^2+c_2^2+c_3^2}{4l^2(nlt+k_1)^{\frac{8-2n}{n}}}. \tag{5.5}$$

The expansion scalar θ and shear scalar σ^2 are given by

$$\theta=\frac{4l}{nlt+k_1}\ \text{and,}\ \sigma^2=\frac{c_1^2+c_2^2+2c_3^2}{2(nlt+k_1)^{\frac{8}{n}}}. \tag{5.6}$$

VI. FIVE DIMENSIONAL NON- SINGULAR MODEL WHEN

In this section we study the five dimensional model of universe for $n=0$

For this non- singular model average scale factor a give as $a=k_2\exp(lt)$

Here the metric coefficients take the form

$$A=d_1k_2\exp(lt)\exp\left[-\frac{c_1\exp(-4lt)}{4lk_2^4}\right], \tag{6.1}$$

$$B=d_2k_2\exp(lt)\exp\left[-\frac{c_2\exp(-4lt)}{4lk_2^4}\right], \tag{6.2}$$

$$C=d_3k_2\exp(lt)\exp\left[-\frac{c_3\exp(-4lt)}{4lk_2^4}\right]. \tag{6.3}$$

The mean generalized Hubble parameter becomes

$$H = l ,\tag{6.4}$$

While the volume scale factor turns out to be

$$V = k_2^4 \exp(4lt)\tag{6.5}$$

The mean anisotropy parameter A becomes

$$A = \left[\frac{c_1^2 + c_2^2 + 2c_3^2}{4l^2 k_2^8}\right] \exp(-8lt),\tag{6.6}$$

While the scalar θ and shear scalar σ^2 are given by

$$\theta = 4l \text{ and } \sigma^2 = \left[\frac{c_1^2 + c_2^2 + 2c_3^2}{2k_2^8}\right] \exp(-8lt)\tag{6.7}$$

VII. CONCLUSION

In this paper, we have observed that the expansion of universe and obtained two five dimensional exact solutions of Bianchi type – I space time in theory of gravity using assumption of constant deceleration parameter and variation law of Hubble parameter. These solutions lead to two different models of the universe. The first solution represents to a singular model for with power law expansion and second solution gives a non-singular model for with exponential expansion of the universe.

VIII. REFERENCES

[1] . Ratbay Myrzakulov: Accelerating universe in f(T) gravity. Eur.Phy.J.C(2011)71:1752

[2] . M. Sharif and Shamaila Rani:f(T) Models within Bianchi Type-I Universe.arXiv:1105.6228v1 [gr-qc] 31 May 2011.

[3] . Wei H. et al:f(T) theories and varying fine structure constant. Phys.Lett.B 703 (2011) 74-80.

[4] . Bamba K. et al: Equation of state for dark energy in f(T) gravity. JACP 1101 (2011) 021.

[5] . Copeland, E.J., Sami, M. and Tsujikawa, S.: Dynamics of dark energy. Int. J. Mod. Phys. D15 (2006)1753.

[6] . Sahni, V. and Starobinsky, A: 5 Dark Matter and Dark Energy .Int. J. Mod. Phys. D9 (2000)373; Sahni,V.: Lect. Notes. Phys. 653 (2004) 141.

[7] . Sharif, M. and Zubair, M.: Analysis of f(R) theory corresponding to NADE and NHDE.Adv. High Energy Phys. 2013(2013) 790967.

[8] . Sharif, M. and Kausar, H. R.: Effects of f(R) model on the dynamical instability of expansion free gravitational collapse. JCAP 07(2011)022.

[9] . Sharif , M. and Kausar, H.R. :Expansion free fluid evolution and skripkin model in f(R) theory. Int. J. Mod. Phys. D20(2011)2239.

[10] . Sharif, M. and Kausar, H.R.: Gravitational perfect fluid collapse in f(R) gravity.Astrophys. Space Sci. 331(2011)281.

[11] . Sharif, M. and Kausar, H.R.: Effects of f(R) dark energy on dissipative anisotropic collapsing fluid. Mod. Phys. Lett. A25(2010)3299.

[12] . Bamba, K., Nojiri, S., Odintsov, S.D. and Saez-Gomez, D.: Possible antigravity regions in F(R) theory? Phys. Lett. B730(2014)136.

[13] . Bamba, K., Makarenko, A.N., Myagky, A.N., Nojiri, S. and Odintsov, S.D.: Bounce cosmology from F(R) gravity and F(R) bigravity. JCAP 01(2014)008.

[14] . Bamba, K., Nojiri, S. and Odintsov,S.D.: Time-dependent matter instability and star singularity in F(R) gravity. Phys. Lett. B698(2011)451.

[15] . Capozziello, S. and Vignolo, S.: The Cauchy problem for metric- affine F(R)-graviry in the presence of perfect-fluid matter. Int. J. Geom. Meth. Mod.Phys. 8(2011)167.

[16] . Capozziello, S., Darabi, F. and Vernieri, D.:Equivalence between palatini and metric formalisms of f(R) – gravity by divergence free current. Mod.Phys.lett. A26(2011)65.

[17] . Capozziello, S., Laurentis, M.D., Odintsov, S.D. and Stabile, A.: Gravitational radiation in Horava gravity. Phys. Rev. D83(2011)064004

[18] . Elizalde, E., Nojiri, S., Odintsov, S.D. and Saez-Gomez, D.: Unifying inflation with dark energy in modified F(R) Horava - Lifshitz gravity. Eur. Phys. J. C70(2010)351.

[19] . Bamba, K., Geng, C., Nojiri, S. and Odintsov, S.D. : An exponential F(R) dark energy model. Mod. Phys. Lett.A25(2010)900.

[20] . Capozziello, S., Laurentis, M.D., Nojiri, S. and Odintsov, S.D.: f(R) gravity constrained by PPN parameters and stochastic back ground of gravitational waves. Gen. Rel. Grav. 41(2009)2313.

[21] . Harko,T., Lobo, F.S.N., Nojiri, S. and Odintsov, S.D.: f(R,T) gravity. Phys. Rev. D84(2011)024020.

[22] . Myrzakulov, R.: FRW and Bianchi type I cosmology of f-essence Phys. Rev. D84(2012)024020.

[23] . Sharif, M. and Zubair, M.: Thermodynamics in f(R,T) theory of gravity.JCAP 03(2012)028.

[24] . Sharif, M. and Zubair, M.: Cosmology of Holographic and New Agegraphic f(R,T) models. J. Phys. Soc. Jpn. 82(2013)064001.

[25] . Adhav, K.S.: LRS Bianchi type-I cosmological model in f (R,T) theory of gravity.Astro.Phys. Space Sci. 339(2012)365.

[26] . L.S. Ladke.: Five Dimensional Exact solutions of Bianchi Type-I Space-Time in f(R,T) Theory of Gravity.Vol.3, Issue 8(2014)

[27] . Houndjio, M.J.S.: Reconstruction of f(R,T) gravity describing matter dominated and accelerated phases. Int. J. Mod. Phys. D21(2012)1250003.

[28] . Adhav, K.S.: LRS Bianchi type –I cosmological model in f(R,T) theory of gravity.Astrophys. Space Sci. 339(2012)365.

[29] . Reddy, D.R.K., Santikumar, R. and Naidu, R.L.: Bianchi type-III cosmological model in f(R,T) theory of gravity. Astrophys. Space Sci. 342(2012)249.

[30] . Ahmed, N. and Pradhan, A.: Bianchi type- V cosmology in f(R,T) gravity with (T).Int. J. Theor.: Phys. 53(2014)289.

[31] . Naidu, R.L., Reddy, D.R.K., Ramprasad, T. and Rammana, K.V.: Bianchi Type- V bulk viscous string cosmological model in f(R,T) gravity. Astrophys. Space Sci. 348(2013)247.

[32] . Deo S. D. and Borsare Vandana S.: Exact Solutions Of Bianchi Type-I Wet Dark Energy in f(R,T) Gravity. DOI 10.17148/IARJSET.2016.3757.
[33] . KaluzaT.:ZumUnitatsproblem. Der Physik. Sitzpreuss.Akad.Wiss. D33, 966-972(1921).
[34] . Kelin O.: Quantum Theory and Five-Dimensional Theory of Relativity.(In German and English)Z. Phys.37(1926)895-906.
[35] . Wesson P S: A new approach to scale-invariant gravity.AstronAstrophys., 119, 145-152,(1983)
[36] . Wesson P S: An embedding for general relativity with variable rest mass.Gen. Rel. Grav. Vol.16, Issue 2,pp. 193-203, (1984).
[37] . Reddy D.R.K. and N. VenkateswaraRao: Non- existence of higher dimensional anisotropic cosmological model in biometric theory of gravitation. Astrophysics and Space Science, Vol.271, Issue 2, pp 165-169.(2000).
[38] . Lorentz and Petzold: Higher- dimensional Brans-Dicke cosmologies.Gen.Relativ.Grav.17, 1189, (1985).
[39] . Ibanez, Z. and Verdaguer, E.:Radiative isotropic cosmologies with extra dimensions.Phys. Rev. D, 34,1202,(1986).
[40] . Adhav. K.S. et al.: N- dimensional string cosmological model in Brans –Dicke theory of gravitation. Astrophysics and Space Science, 310(3-4), 231, (2007).
[41] . Harko,T., Lobo, F.S.N., Nojiri, S. and Odintsov, S.D.: “ f(R,T) gravity”.Phys. Rev. D84(2011)024020.

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On Strongly Rβgc*-Continuous Mappings in Topological Spaces

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ABSTRACT
The Aim of this paper is to introduce a new type of mappings called strongly Regular β-generalized c*-continuous mappings and study their basic properties. Also, we establish the relationship between strongly Regular β-generalized c*-continuous mappings and other near continuous mappings in topological spaces.
Keywords : Strongly βgc*-closed sets, Strongly βgc*-open sets, Strongly βgc*-continuous mappings, Strongly Rβgc*-continuous mappings.

I. INTRODUCTION

In 1963, Norman Levine introduced semi-open sets in topological spaces. Also in 1970, he introduced the concept of generalized closed sets. N. Levine introduced the concept of semi-continuous function in 1963. In 1980, Jain introduced totally continuous functions. In 1995, T.M. Nour introduced the concept of totally semi-continuous functions as a generalization of totally continuous functions. In 2011, S.S. Benchalli et.al introduced the concept of semi-totally continuous functions in topological spaces. In this paper we introduce Strongly Regular β-generalized c*-continuous mappings in topological spaces and study their basic properties.
Section 2 deals with the preliminary concepts. In section 3, Strongly Regular β-generalized c*-continuous mappings are introduced and their basic properties are studied.

II. PRELIMINARIES

Throughout this paper X denotes a topological space on which no separation axiom is assumed. For any subset A of X, cl(A) denotes the closure of A, int(A) denotes the interior of A. Further X\A denotes the complement of A in X. The following definitions are very useful in the subsequent sections.

Definition: 2.1 A subset A of a topological space X is called

i. a semi-open set [9] if $A\subseteq cl(int(A))$ and a semi-closed set if $int(cl(A))\subseteq A$.
ii. a regular-open set [18] if $A=int(cl(A))$ and a regular-closed set if $A=cl(int(A))$.

iii. a π-open set [19] if A is the finite union of regular-open sets and the complement of π-open set is said to be π-closed.
iv. a β-open set [1] (semi-pre open set[2]) if $A \subseteq cl(int(cl(A)))$ and a β-closed set (semi-pre closed set) if $int(cl(int(A))) \subseteq A$.

Definition: 2.2 A subset A of a topological space X is said to be a clopen set if A is both open and closed in X.

Definition: 2.3 [10] A subset A of a topological space X is said to be a c*-open (semi-clopen) set if $int(cl(A)) \subseteq A \subseteq cl(int(A))$.

Definition: 2.4 [16] A subset A of a topological space X is called a π-generalized β-closed (briefly, $\pi g\beta$-closed) set if $\beta cl(A) \subseteq H$ whenever $A \subseteq H$ and H is π-open in X. The complement of the $\pi g\beta$-closed set is said to be $\pi g\beta$-open.

Definition: 2.5 [13] A subset A of a topological space X is called a generalized semi pre regular-closed (briefly, gspr-closed) set if $spcl(A) \subseteq H$ whenever $A \subseteq H$ and H is regular-open in X. The complement of the gspr-closed set is said to be gspr-open.

Definition: 2.6 [10] A subset A of a topological space X is said to be a generalized c*-closed set (briefly, gc*-closed set) if $cl(A) \subseteq H$ whenever $A \subseteq H$ and H is c*-open. The complement of the gc*-closed set is gc*-open [11].

Definition: 2.7 [12] A subset A of a topological space X is said to be strongly β-generalized c*-closed (briefly, strongly βgc*-closed) if $\beta cl(A) \subseteq H$ whenever $A \subseteq H$ and H is gc*-open in X. The complement of the strongly βgc*-closed set is said to be strongly βgc*-open.

Definition: 2.8 A function $f : X \rightarrow Y$ is called
i. Semi-continuous [9] if the inverse image of every open subset of Y is semi-open in X.
ii. Totally continuous [7] if the inverse image of every open subset of Y is clopen in X.
iii. Strongly continuous [8] if the inverse image of every subset of Y is clopen in X.
iv. Totally semi-continuous [15] if the inverse image of every open subset of Y is semi- clopen in X.
v. Strongly semi-continuous [15] if the inverse image of every subset of Y is semi- clopen in X.
vi. Semi-totally continuous [3] if the inverse image of every semi- open subset of Y is clopen in X.
vii. Semi-totally semi-continuous [6] if the inverse image of every semi-open subset of Y is semi-clopen in X.
viii. S-continuous [14] if the inverse image of every semi-open subset of Y is open in X.
ix. Almost-continuous [17] if the inverse image of every regular-open subset of Y is open in X.
x. Regular set connected [4] if the inverse image of every regular-open subset of Y is clopen in X.
xi. Π-generalized β-continuous [16] (briefly, $\pi g\beta$-continuous) if the inverse image every closed set in Y is $\pi g\beta$-closed in X.
xii. Generalized semipre regular-continuous [13] (briefly, gspr-continuous) if the inverse image of every closed

set in Y is gspr-closed in X.

Definition: 2.9 [5] A space X is said to be locally indiscrete if every closed set is regular closed in X.

III. STRONGLY REGULAR β-GENERALIZED C*-CONTINUOUS MAPPINGS

In this section, we introduce strongly Regular β-generalized c*-continuous mappings and study their relation with near continuous mappings.

Definition: 3.1 A mapping $f:X\rightarrow Y$ is said to be strongly Regular βgc*-continuous (briefly, strongly Rβgc*-continuous) if the inverse image of every regular-closed set in Y is strongly βgc*-closed in X.

Example: 3.2 Let X={a,b,c,d} with topology τ={∅, {a}, {c}, {d}, {a,c}, {a,d}, {c,d}, {a,c,d}, X} and Y={1,2,3,4} with topology σ={∅, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {3,4}, {1,2,3}, {2,3,4}, {1,3,4}, Y}. Define $f : X \rightarrow Y$ by f(a)=2, f(b)=4, f(c)=3, f(d)=1. Then the inverse image of every regular-closed set in Y is strongly βgc*-closed in X. Hence f is strongly Rβgc*-continuous.

Proposition: 3.3 Every continuous mapping is strongly Rβgc*-continuous.
Proof: Assume that $f:X\rightarrow Y$ is continuous. Let V be a regular-closed set in Y. Then V is closed in Y. Since f is continuous, $f^{-1}(V)$ is closed in X. Therefore, by Proposition 3.3 [12], $f^{-1}(V)$ is strongly βgc*-closed in X. Hence f is strongly Rβgc*-continuous.

Proposition: 3.4 Every semi-continuous mapping is strongly Rβgc*-continuous.
Proof: Assume that $f:X\rightarrow Y$ is semi-continuous. Let V be a regular-closed set in Y. Then V is closed in Y. Since f is semi-continuous, $f^{-1}(V)$ is semi-closed in X. Therefore, by Proposition 3.7[12], $f^{-1}(V)$ is strongly βgc*-closed in X. Hence f is strongly Rβgc*-continuous.

Proposition: 3.5 Every totally continuous mapping is strongly Rβgc*-continuous.
Proof: Assume that $f:X\rightarrow Y$ is totally continuous. Let V be a regular-closed set in Y. Then V is closed in Y. Since f is totally continuous, $f^{-1}(V)$ is clopen in X. This implies, $f^{-1}(V)$ is closed in X. Therefore, by Proposition 3.3[12], $f^{-1}(V)$ is strongly βgc*-closed in X. Hence f is strongly Rβgc*-continuous.

Proposition: 3.6 Every totally semi-continuous mapping is strongly Rβgc*-continuous.
Proof: Assume that $f:X\rightarrow Y$ is totally semi-continuous. Let V be a regular-closed set in Y. Then V is closed in Y. Since f is totally semi-continuous, $f^{-1}(V)$ is semi-clopen in X. This implies, $f^{-1}(V)$ is semi-closed in X. Therefore, by Proposition 3.7[12], $f^{-1}(V)$ is strongly βgc*-closed in X. Hence f is strongly Rβgc*-continuous.

Proposition: 3.7 Every strongly semi- continuous mapping is strongly Rβgc*-continuous.

Proof: Assume that $f:X\to Y$ is strongly semi-continuous. Let V be a regular-closed set in Y. Then V is closed in Y. Since f is strongly semi-continuous, $f^{-1}(V)$ is semi-clopen in X. This implies, $f^{-1}(V)$ is semi-closed in X Therefore, by Proposition 3.7[12], $f^{-1}(V)$ is strongly βgc*-closed in X. Hence f is strongly Rβgc*-continuous.

Proposition: 3.8 Every strongly continuous mapping is strongly Rβgc*-continuous.
Proof: Assume that $f:X\to Y$ is strongly continuous. Let V be a regular-closed set in Y. Then V is closed in Y. Since f is strongly continuous, $f^{-1}(V)$ is clopen in X. This implies, $f^{-1}(V)$ is closed in X. Therefore, by Proposition 3.3[12], $f^{-1}(V)$ is strongly βgc*-closed in X. Hence f is strongly Rβgc*-continuous.

Proposition: 3.9 Every semi-totally continuous mapping is strongly Rβgc*-continuous.
Proof: Assume that $f:X\to Y$ is semi-totally continuous. Let V be a regular-closed set in Y. Then V is semi-closed in Y. Since f is semi- totally continuous, $f^{-1}(V)$ is clopen in X. This implies, $f^{-1}(V)$ is closed in X Therefore, by Proposition 3.3[12], $f^{-1}(V)$ is strongly βgc*-closed in X. Hence f is strongly Rβgc*-continuous.

Proposition: 3.10 Every semi-totally semi-continuous mapping is strongly Rβgc*-continuous.
Proof: Assume that $f:X\to Y$ is semi-totally semi- continuous. Let V be a regular-closed set in Y. Then V is semi-closed in Y. Since f is semi-totally semi-continuous, $f^{-1}(V)$ is semi-clopen in X. This implies, $f^{-1}(V)$ is semi-closed in X. Therefore, by Proposition 3.7[12], $f^{-1}(V)$ is strongly βgc*-closed in X. Hence f is strongly Rβgc*-continuous.

Proposition: 3.11 Every s-continuous mapping is strongly Rβgc*-continuous.
Proof: Assume that $f:X\to Y$ is s-continuous. Let V be a regular- closed set in Y. Then V is semi-closed in Y. Since f is s-continuous, $f^{-1}(V)$ is closed in X. Therefore, by Proposition 3.3[12], $f^{-1}(V)$ is strongly βgc*-closed in X. Hence f is strongly Rβgc*-continuous.

Proposition: 3.12 Every almost-continuous mapping is strongly Rβgc*-continuous.
Proof: Assume that $f:X\to Y$ is almost-continuous. Let V be a regular-closed set in Y. Then $f^{-1}(V)$ is closed in X. Therefore, by Proposition 3.3[12], $f^{-1}(V)$ is strongly βgc*-closed in X. Hence f is strongly Rβgc*-continuous.

Proposition: 3.13 Every Regular set-connected mapping is strongly Rβgc*-continuous.
Proof: Assume that $f:X\to Y$ is Regular set-connected. Let V be a regular-closed set in Y. Then $f^{-1}(V)$ is clopen in X. This implies, $f^{-1}(V)$ is closed in X Therefore, by Proposition 3.3[12], $f^{-1}(V)$ is strongly βgc*-closed in X. Hence f is strongly Rβgc*-continuous.
The Converse of the above Propositions need not be true as shown in the following example.

Example:3.14 Let $X=\{a,b,c,d,e\}$ with topology $\tau=\{\emptyset,\{a,b\},\{c,d\},\{a,b,c,d\},X\}$ and $Y=\{1,2,3,4,5\}$ with topology $\sigma=\{\emptyset,\{1\},\{2\},\{1,2\},\{1,2,3\}, \{1,2,3,4\},\{1,2,3,5\},Y\}$. Define $f:X\to Y$ by $f(a)=1$, $f(b)=2$, $f(c)=3$, $f(d)=4$, $f(e)=5$. Then f is strongly Rβgc*-continuous. But f is not continuous (semi-continuous, totally semi-continuous, totally continuous, semi-totally continuous, semi-totally semi-continuous, strongly continuous, strongly semi-continuous, s-continuous, almost-continuous, regular set-connected), since $f^{-1}(\{1\})=\{a\}$.

Proposition: 3.15 Every strongly βgc*-continuous mapping is strongly Rβgc*-continuous.

Proof: Assume that f:X→Y is strongly βgc*-continuous. Let V be a regular-closed set in Y. Then V is closed in Y. Since f is strongly βgc*-continuous, $f^{-1}(V)$ is strongly βgc*-closed in X. Hence f is strongly Rβgc*-continuous.

The Converse of the above Proposition need not be true as shown in the following example.

Example: 3.16 Let X={a,b,c,d,e} with topology τ={∅,{a,b},{c,d},{a,b,c,d},X} and Y={1,2,3,4,5} with topology σ={∅,{1},{2},{1,2},{1,2,3}, {1,2,3,4},{1,2,3,5},Y}. Define g:X→Y by g(a)=g(b)=g(c)=g(d)=5, g(e)=4. Then g is strongly Rβgc*-continuous. But g^{-1}({5})={a,b,c,d} which is not a strongly βgc*-closed set in X. Therefore, g is not strongly βgc*-continuous.

Proposition: 3.17 Let X be a topological space and Y be a locally indiscrete space. Then every strongly Rβgc*-continuous mapping f:X→Y is πgβ-continuous.

Proof: Assume that f:X→Y is strongly Rβgc*-continuous. Let V be a closed set in Y. Then V is regular-closed in Y, since Y is locally indiscrete. Since f is strongly Rβgc*-continuous, $f^{-1}(V)$ is strongly βgc*-closed in X. Therefore, by Proposition 3.12 [12], $f^{-1}(V)$ is πgβ-closed in X. Hence f is πgβ-continuous.

Proposition: 3.18 Let X be a topological space and Y be a locally indiscrete space. Then every strongly Rβgc*-continuous mapping f:X→Y is gspr-continuous.

Proof: Assume that f:X→Y is strongly Rβgc*-continuous. Let V be a closed set in Y. Then V is regular-closed in Y, since Y is locally indiscrete. Since f is strongly Rβgc*-continuous, $f^{-1}(V)$ is strongly βgc*-closed in X. Therefore, by Proposition 3.13 [12], $f^{-1}(V)$ is gspr-closed in X. Hence f is gspr-continuous.

Proposition: 3.19 The mapping f:X→Y is strongly Rβgc*-continuous if and only if the inverse image of every regular-open set in Y is strongly βgc*-open in X.

Proof: Assume that f:X→Y is strongly Rβgc*-continuous. Let U be a regular-open set in Y. Then Y\U is regular-closed in Y. This implies, f^{-1}(Y\U) is strongly βgc*-closed in X. Since f^{-1}(Y\U)=X\$f^{-1}$(U), we have X\$f^{-1}$(U) is strongly βgc*-closed in X. This implies, f^{-1}(U) is strongly βgc*-open in X. Conversely, assume that f^{-1}(U) is strongly βgc*-open in X for every regular-open set U in Y. Let V be a regular-closed set in Y. Then Y\V is regular-open in Y. Therefore, f^{-1}(Y\V) is strongly βgc*-open in X. That is, X\$f^{-1}$(V) is strongly βgc*-open in X. This implies, f^{-1}(V) is strongly βgc*-closed in X. Therefore, f is strongly Rβgc*-continuous.

Remark: 3.20 Composition of two strongly Regular βgc*-continuous mappings need not be strongly Regular βgc*-continuous. For example, let X={a,b,c,d}, Y={1,2,3,4}, Z={p,q,r,s,t}. Then, clearly τ={∅,{a},{b},{a,b},{a,c}, {a,b,c},X} is a topology on X, σ={∅, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {3,4}, {1,2,3}, {2,3,4}, {1,3,4},Y} is a topology on Y and η={∅,{p}, {s},{t},{p,s},{p,t},{s,t}, {p,s,t},Z} is a topology on Z. Define f : X → Y by f(a)= f(d)=1, f(b)= f(c)=2 and g : Y → Z by g(1)=p, g(2)=g(3)= g(4)=t. Then f and g are strongly Rβgc*-continuous. Consider the regular-closed set {p,q,r} in Z. Then $(g\circ f)^{-1}$({p,q,r})= $f^{-1}(g^{-1}$({p,q,r}))=f^{-1}({1})={a,d}, which is not a strongly βgc*-closed set in X. Therefore, g∘f is not strongly Rβgc*-continuous.

Proposition: 3.21 If f:X→Y is strongly βgc*-continuous and g:Y→Z is continuous, then g∘f:X→Z is strongly Rβgc*-continuous.

Proof: Assume that f:X→Y is strongly βgc*-continuous and g:Y→Z is continuous. Let V be a regular-closed set in Z. Then $g^{-1}(V)$ is closed in Y. Therefore, $f^{-1}(g^{-1}(V))$ is strongly βgc*-closed in X. That is, $(g\circ f)^{-1}(V)$ is strongly βgc*-closed in X. Hence g∘f is strongly Rβgc*-continuous.

Proposition: 3.22 If f:X→Y is strongly βgc*-continuous and g:Y→Z is totally continuous, then g∘f:X→Z is strongly Rβgc*-continuous.

Proof: Assume that f:X→Y is strongly βgc*-continuous and g:Y→Z is totally-continuous. Let V be a regular-closed set in Z. Then $g^{-1}(V)$ is clopen in Y. This implies, $g^{-1}(V)$ is closed in Y. Therefore, $f^{-1}(g^{-1}(V))$ is strongly βgc*-closed in X. That is, $(g\circ f)^{-1}(V)$ is strongly βgc*-closed in X. Hence g∘f is strongly Rβgc*-continuous.

Proposition: 3.23 If f:X→Y is strongly βgc*-continuous and g:Y→Z is strongly continuous, then g∘f:X→Z is strongly Rβgc*-continuous.

Proof: Assume that f:X→Y is strongly βgc*-continuous and g:Y→Z is strongly continuous. Let V be a regular-closed set in Z. Then $g^{-1}(V)$ is clopen in Y. This implies, $g^{-1}(V)$ is closed in Y. Therefore, $f^{-1}(g^{-1}(V))$ is strongly βgc*-closed in X. That is, $(g\circ f)^{-1}(V)$ is strongly βgc*-closed in X. Hence g∘f is strongly Rβgc*-continuous.

Proposition: 3.24 If f:X→Y is strongly βgc*-continuous and g:Y→Z is semi-totally continuous, then g∘f:X→Z is strongly Rβgc*-continuous.

Proof: Assume that f:X→Y is strongly βgc*-continuous and g:Y→Z is semi-totally continuous. Let V be a regular-closed set in Z. Then $g^{-1}(V)$ is clopen in Y. This implies, $g^{-1}(V)$ is closed in Y. Therefore, $f^{-1}(g^{-1}(V))$ is strongly βgc*-closed in X. That is, $(g\circ f)^{-1}(V)$ is strongly βgc*-closed in X. Hence g∘f is strongly Rβgc*-continuous.

Proposition: 3.25 If f:X→Y is strongly Rβgc*-continuous and g:Y→Z is totally continuous, then g∘f:X→Z is strongly Rβgc*-continuous.

Proof: Assume that f:X→Y is strongly Rβgc*-continuous and g:Y→Z is totally-continuous. Let V be a regular-closed set in Z. Then $g^{-1}(V)$ is clopen in Y. This implies, $g^{-1}(V)$ is regular-closed in Y. Therefore, $f^{-1}(g^{-1}(V))$ is strongly βgc*-closed in X. That is, $(g\circ f)^{-1}(V)$ is strongly βgc*-closed in X. Hence g∘f is strongly Rβgc*-continuous.

Proposition: 3.26 If f:X→Y is strongly Rβgc*-continuous and g:Y→Z is strongly continuous, then g∘f:X→Z is strongly Rβgc*-continuous.

Proof: Assume that f:X→Y is strongly Rβgc*-continuous and g:Y→Z is strongly continuous. Let V be a regular-closed set in Z. Then $g^{-1}(V)$ is clopen in Y. This implies, $g^{-1}(V)$ is regular-closed in Y. Therefore, $f^{-1}(g^{-1}(V))$ is strongly βgc*-closed in X. That is, $(g\circ f)^{-1}(V)$ is strongly βgc*-closed in X. Hence g∘f is strongly Rβgc*-continuous.

Proposition: 3.27 If f:X→Y is strongly Rβgc*-continuous and g:Y→Z is semi-totally continuous, then g∘f:X→Z is strongly Rβgc*-continuous.

Proof: Assume that f:X→Y is strongly Rβgc*-continuous and g:Y→Z is semi-totally continuous. Let V be a regular-closed set in Z. Then $g^{-1}(V)$ is clopen in Y. This implies, $g^{-1}(V)$ is regular-closed in Y. Therefore, $f^{-1}(g^{-1}(V))$ is strongly βgc*-closed in X. That is, $(g∘f)^{-1}(V)$ is strongly βgc*-closed in X. Hence g∘f is strongly Rβgc*-continuous.

Proposition: 3.28 If f:X→Y is strongly Rβgc*-continuous and g:Y→Z is totally continuous, then g∘f:X→Z is strongly βgc*-continuous.

Proof: Assume that f:X→Y is strongly Rβgc*-continuous and g:Y→Z is totally-continuous. Let V be a closed set in Z. Then $g^{-1}(V)$ is clopen in Y. This implies, $g^{-1}(V)$ is regular-closed in Y. Therefore, $f^{-1}(g^{-1}(V))$ is strongly βgc*-closed in X. That is, $(g∘f)^{-1}(V)$ is strongly βgc*-closed in X. Hence g∘f is strongly βgc*-continuous.

Proposition: 3.29 If f:X→Y is strongly Rβgc*-continuous and g:Y→Z is strongly continuous, then g∘f:X→Z is strongly βgc*-continuous.

Proof: Assume that f:X→Y is strongly Rβgc*-continuous and g:Y→Z is strongly continuous. Let V be a closed set in Z. Then $g^{-1}(V)$ is clopen in Y. This implies, $g^{-1}(V)$ is regular-closed in Y. Therefore, $f^{-1}(g^{-1}(V))$ is strongly βgc*-closed in X. That is, $(g∘f)^{-1}(V)$ is strongly βgc*-closed in X. Hence g∘f is strongly βgc*-continuous.

Proposition: 3.30 If f:X→Y is strongly Rβgc*-continuous and g:Y→Z is semi-totally continuous, then g∘f:X→Z is strongly βgc*-continuous.

Proof: Assume that f:X→Y is strongly Rβgc*-continuous and g:Y→Z is semi-totally continuous. Let V be a closed set in Z. This implies, V is semi-closed in Z. Therefore, $g^{-1}(V)$ is clopen in Y. This implies, $g^{-1}(V)$ is regular-closed in Y. Therefore, $f^{-1}(g^{-1}(V))$ is strongly βgc*-closed in X. That is, $(g∘f)^{-1}(V)$ is strongly βgc*-closed in X. Hence g∘f is strongly βgc*-continuous.

IV. CONCLUSION

In this paper we have introduced strongly Regular β-generalized c*-continuous mappings in topological spaces and studied some of their basic properties. Also, we have discussed the relation of strongly Regular β-generalized c*-continuous mappings with near continuous mappings in topological spaces.

V. REFERENCES

[1]. M.E. Abd El-Monsef, S.N. El-Deeb and R.A. mahmoud, "β-open sets and β-continuous mappings", Bull. Fac. Sci. Assiut univ. 12(1983), 77-90.
[2]. D. Andrijevic, "Semi pre open sets", Mat. Vesnik, 38(1986), 24-32.
[3]. S.S. Benchalli and U. I Neeli, "Semi-totally Continuous function in topological spaces", Inter. Math. Forum, 6 (2011), 10, 479-492.
[4]. J. Dontchev, M. Ganster and I. L. Reilly, "More on almost s-continuity", Indian Journal of Mathematics, 41 (1999), 139-146.

[5] . J. Dontchev, Survey on Pre-open Sets, "The proceedings of the Yatsushiro Topological Conference", (1998), 1-18.

[6] . Hula M salih, "Semi-totally Semi-continuous functions in topological spaces", AL-Mustansiriya university college of Education, Dept. of Mathematics.

[7] . R.C. Jain, "The role of regularly open sets in general topological spaces", Ph.D. thesis, Meerut University, Institute of advanced studies, Meerut-India, (1980).

[8] . N. Levine, "Strong continuity in topological space, Amer. Math. Monthly"., 67 (1960), 269.

[9] . N. Levine, "Semi-open sets and semi-continuity in topological space, Amer. Math. Monthly'., 70 (1963), 39-41.

[10] . S. Malathi and S. Nithyanantha Jothi, "On c*-open sets and generalized c*-closed sets in topological spaces", Acta ciencia indica, Vol. XLIIIM, No.2, 125 (2017), 125-133.

[11] . S. Malathi and S. Nithyanantha Jothi, "On generalized c*-open sets and generalized c*-open maps in topological spaces", Int. J. Mathematics And its Applications, Vol. 5, issue 4-B (2017), 121-127.

[12] . S. Malathi and J. Maheswari, "On Strongly β-generalized c*-closed sets in topological spaces, International Journal of Mathematics Trends and Technology", Vol. 67 Issue 6, (2021), 190-194.

[13] . G. B. Navalagi, A.S. Chandrashekarappa and S.V. Gurushantanavar, On "gspr closed sets in topological spaces, International Journal of Mathematics and Computer Applications", Vol. 2, No., 1-2, pp. 51-58, 2010.

[14] . T. Noiri, B. Ahmad and M. Khan, '"Almost S-continuous functions", Kyungpook Math. Journal, Vol. 35 (1995), 311-322.

[15] . T. M. Nour, (1995), "Totally semi-continuous function", Indian J. Pure Appl.Math.,7, 26, 675-678.

[16] . S. Tahiliani, "On πgβ-closed Sets in topological spaces", Note di Mathematica, Vol. 30 (1) (2010), 49-55.

[17] . M. K. Singal and A. R. Singal, "Almost continuous mappings", Yokohama Math. .,Vol. 16 (1968), 63-73.

[18] . M. Stone, "Application of the theory of Boolean rings to general topology", Trans. Amer. Math. Soc., 41(1937), 374-481.

[19] . V. Zaitsav, "On Certain classes of topological spaces and their bicompactifications", Dokl. Akad. Nauk. SSSR, 178(1968), 778-779.

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Parameter Analysis of "ATM Model"

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ABSTRACT

Mathematics is a subject which is still a nightmare for almost every student. The reason behind this is not just the subject but also students' least interest in self-learning. Now a days students are totally dependent on teachers; they do not want to put their efforts into the subject. In this paper the authors have discussed an experiment using "ATM Model" that was done on their B.Sc. students to create interest of the subject in students and to make the students self-learn Mathematics.

Keywords: ATM, Facilitator, Self-learner, GST

I. INTRODUCTION

If we ask students about the subject Mathematics then majority will answer that it is very tough subject. The reason behind this answer is not that the subject is actually very tough but one of the main reasons is that students do not want to put their own efforts into the subject due to lack of self-study.

It is observed that students are now a days very less interested in self-learning subjects, especially Mathematics. This results in low passing percentage which creates a fear about the subject among them. In the process of teaching-learning-evaluation, most of the time, the learning part is neglected by both teachers and students, thus affecting evaluation of the subject in general and of individuals, in particular.

"ATM Model" which means **"Any Time Mathematics with Assistance Teacher Mentors Model"** is an attempt to make learning of Mathematics more interesting and self-explanatory rather than the one-sided monotonous teaching. This Model does not require many teachers, which is cost effective. This Model can be used in engaging students in absence of sufficient number of teachers in a department and can be applied to other subjects also. This Model will work for every department.

The method using the "ATM Model" was experimented on B.Sc. Semesters II, IV and VI students of Hislop College, Nagpur for the session 2017-2018. In this paper the authors have discussed the method that was experimented on the students and the results that were observed.

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II. METHOD

From each semester, in a class of more than 100 students, 16 students willingly volunteered themselves on the basis of their interest, intelligence, aptitude, motivation, self-concept and readiness for mathematics. These career-oriented students having qualities of love, affection, dedication and pursuing their career in Mathematics were named the **"Assistant Teacher Mentors (ATM)"**.

The 16 ATMs were divided into 8 pairs by the teachers. Whole syllabi consisting of 8 units were divided among them in such a way that each pair got 1 unit. They were given 15 days' time to do self-study of their respective units and prepare the notes. After rigorous self-study, each pair prepared their own study materials.

The teachers, under the supervision of Head of the Department, thoroughly went through the study materials prepared by each pair of ATMs and made the necessary corrections. After that, the study materials were printed in the form of a Handbook cum Workbook with ISBN which was by the students and for the students. This way in total three Handbooks cum Workbooks with ISBN 978-81-934998-1-8, 978-81-934998-2-5, 978-81-934998-3-2 were ready for all the semesters II, IV and VI respectively (Figure 1, 2, 3)

Figure 1: B.Sc. Semester II

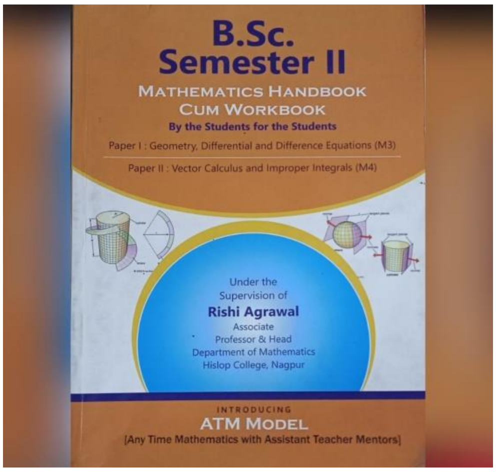


Figure 2: B.Sc. Semester IV

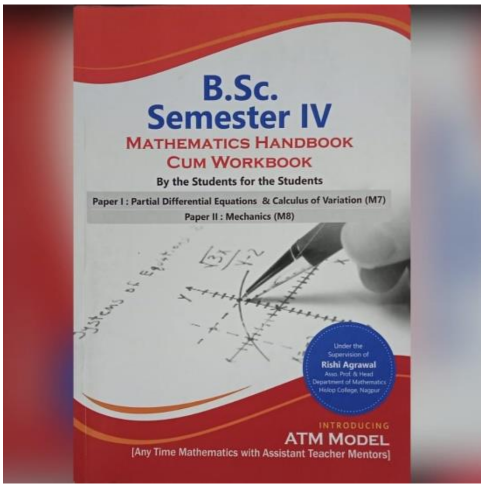
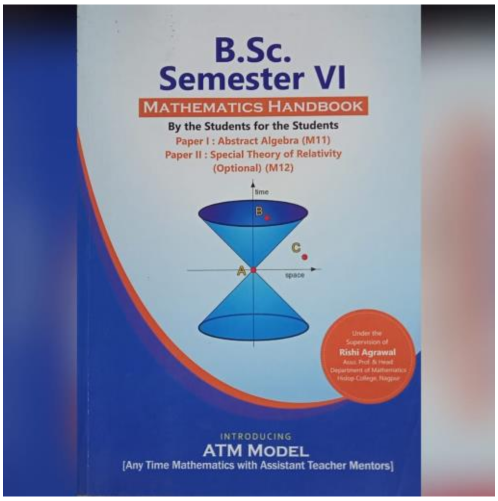


Figure 3: B.Sc. Semester VI



The books have every left side printed with the study materials and the right side empty so that students (other than the ATMs) could self-study the materials from the printed side and then write the same material on the empty side according to their own understandings. These books were printed only for students of the Department of Mathematics, Hislop College, Nagpur. The cost of printing was collected from the students and all the copies were distributed among them which makes these books **“By the students and for the students”.**

During regular classes, sitting arrangements were made twice a week for the students in such a way that the whole class was divided into groups using the concept of Modulo 8 (Table 1), such that each group contain an ATM.

Table 1: Modulo 8 Classes

Sr. No.								
1	1	2	3	4	5	6	7	8
2	9	10	11	12	13	14	15	16
3	17	18	19	20	21	22	23	24
4	25	26	27	28	29	30	31	32
5	33	34	35	36	37	38	39	40
6	41	42	43	44	45	46	47	48
7	49	50	51	52	53	54	55	56
8	57	58	59	60	61	62	63	64
9	65	66	67	68	69	70	71	72
10	73	74	75	76	77	78	79	80
11	81	82	83	84	85	86	87	88
12	89	90	91	92	93	94	95	96
13	97	98	99	100	101	102	103	104
14	105	106	107	108	109	110	111	112
15	113	114	115	116	117	118	119	120
16	121	122	123	124	125	126	127	128

17	129	130	131	132	133	134	135	136
18	137	138	139	140	141	142	143	144
19	145	146	147	148	149	150	151	152
20	153	154	155	156	157	158	159	160

The ATMs helped the students of their respective group in understanding doubts from their assigned units. The groups were formed in such a way that all the students got an opportunity to understand all the 8 units with the help of the ATMs. Also, the ATM's were given a chance to share their learning experience with others and thus motivate them for self- learning under the supervision of teachers. The students who shared their time in this self-learning process named as "**Gained by Sharing Time (GST)".**

III. RESULTS AND DISCUSSION

The parameters involved in the experiment using ATM Model are:

i. Dedicated ATMs,

ii. Teachers in charge,

iii. Students with the benefit of GST

We must note that, it takes almost three months for at least two teachers to teach all the 8 units. However, this Model helped in creating study materials of all 8 units in just 15 days and another 1 month for students to learn these units. So, This Model took half time to complete the syllabus of Mathematics.

This Model created an environment of Group study and developed a nature of helping each other. Majority of the students generally do not come to the teachers to clear their doubts in some topic but with the help of this method, students felt free to ask their doubts to the ATMs who were their peers, which also helped students to clear their doubts and learn Mathematics with good understanding.

The main achievement of this Model was to make students enjoy learning Mathematics by making it easy for all the students by encouraging them to develop the habit of self-study, which also prepared the students for higher studies and research.

IV. CONCLUSIONS

Mathematics is a subject through which one can develop many skills and qualities, like imagination, patience, self-esteem, analysis, time management, confidence, honesty, goal setting, problem solving & decision making, team building & leadership skills etc. With the fundamentals of Mathematics, one can stand in the applied and scientific fields too. Understanding or learning Mathematics is doing Mathematics.

Hence "Any Time Mathematics with Assistance Teacher Mentors Model" viz. **"ATM Model"** makes learning Mathematics enjoyable for students. In this method, the teachers play the role of the **FACILITATORS** and students become **SELF LEARNERS** and take the benefits of **"GST".**

This Model is cost effective, engage students with a smaller number of teachers, by the students for the students and can be used by any department to make the subject easy and self-learning for students.

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VI.REFERENCES

[1] . R. K. Agrawal : “GANEET DARSHAN – At a Glance”, Proceedings of the National Conference ‘Signposting NAAC: New Directions for Quality Assurance & Quality Sustenance in Accredited / Reaccredited Higher Education Institutions’, 14-15 Dec. 2012, pp 103-108.

[2] . R. K. Agrawal, S. R. Dehri : “Integration Through Charity”, IJSRSET, Vol. 9 – Issue 6, Sep10, 2021, pp 1-2.

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